A Generalized Random Mobility Model for Wireless Ad Hoc Networks and its Analysis: One-Dimensional Case

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***Abstract*— In wireless ad hoc networks, the ability to analytically characterize the spatial distribution of terminals plays a key role in understanding fundamental network QoS measures such as throughput per source to destination pair, probability of successful transmission, connectivity, etc. Consequently, mobility models that are general enough to capture the major characteristics of a realistic movement profile, and yet are simple enough to mathematically formulate its long-run behavior, are highly desirable.**

In this paper, we propose a generalized random mobility model capable of capturing several mobility scenarios and give a mathematical framework for its exact analysis over one-dimensional mobility terrains. The model provides the flexibility to capture hotspots where mobiles accumulate with higher probability and spend more time. The selection process of hotspots is random and correlations between the consecutive hotspot decisions can be successfully modeled. Furthermore, the times spent at the destinations can be dependent on the location of destination point, the speed of movement can be a function of distance that is being traveled, and the acceleration characteristics of vehicles can be incorporated into the model. Our solution framework formulates the model as a semi-Markov process using a special discretization technique. We provided long-run location and speed distributions by closed-form expressions for one-dimensional regions (e.g., a highway).

***Index Terms*— Mobility Modeling, Long-Run Analysis, semi-Markov Processes, Ad Hoc Networks**

I. Introduction

WIRELESS ad hoc networks are comprised of wireless mobile nodes that can dynamically form a network in a self-organizing manner without the need for a pre-existing fixed infrastructure. Nodes in an ad hoc network can move according to many different mobility profiles. Therefore, mobility models that dictate the movement behavior of a mobile terminal play a key role in analyzing the impact of dynamically changing topology on the performance of these networks, which can be done through analytical or simulation based studies. In this paper, we consider a generalized random mobility model that is flexible enough to capture different mobility scenarios, and provide its long-run location and speed distributions by closed form expressions for one-dimensional mobility terrains.

In what follows, we categorize the existing mobility models for wireless ad hoc networks, and briefly summarize their assumptions. Traditionally, a mobility model governs the changes in the moving direction and speed of terminals according to a deterministic approach or a random process. In the former case, movement path of terminals can be restricted to predetermined paths. For ad hoc environments, such mobility models are impractical since wireless ad hoc networks are created “on the fly”, and collecting data to generate the paths for all situations can be very complicated. Thus, a mobility model that dictates the movement of hosts due to a random process, that is, *random mobility model*, is more appropriate for the performance evaluation of these networks. Surveys for both models are presented in [1], [2].

In general, random mobility models formulate the movement pattern of mobile hosts by consecutive random length intervals called movement epochs. During each epoch, mobile terminal moves at a constant speed, and at a constant direction for a random amount of time. The speed and direction choice for each epoch may or may not be correlated with their values in the previous epochs, and mobility characteristics of other terminals. For instance, according to the random walk mobility model [2], each terminals movement is uncorrelated with others movement, and the speed and direction choices for each epoch are also uncorrelated with their previous choices. The random waypoint mobility model [3] includes pauses at the end of movement epochs in the random walk model to make it more applicable to different scenarios. More formally, according to the random waypoint mobility model, a mobile node determines a destination point that is distributed *uniformly* within the physical terrain and moves in the direction of that destination at a constant speed. This speed is selected uniformly from $[v_{\min}, v_{\max}]$ where $v_{min} > 0$, and it is *independent* from the destination and starting points of the movement epoch, and also the distance that is going to be traveled. After reaching the destination, mobile pauses for a random amount of time, which has the *same distribution* for all destination points, and the same movement process is repeated by selecting a new destination and speed pair *independently* from the same pair of the previous movement epoch.

A shortcoming of the random mobility models is that the movement profiles that are generated with respect to them may not be consistent with the major characteristics of a realistic scenario. For instance, as it also mentioned in [1], random walk and random waypoint mobility like models may generate unrealistic movement patterns such as "sudden stops" and "sharp turns". In [4], [5], [6], authors propose models that can capture correlation between the speed and the direction choices of consecutive movement epochs and therefore these models may generate a pattern which is smoother with less sharp turns. Furthermore, as it is also criticized in [7], [8], selecting speed independently from the distance that is going to be traveled may end up in unrealistic mobility profiles where mobiles travel long distances with low speeds.

The common limitation of the random mobility models described above is that one can not model a scenario which incorporates predefined pathways that mobiles must follow and specific destinations on those paths where mobiles accumulate with higher probability. The models presented in [9], [10] focuses on this problem by taking a more deterministic approach that can capture obstacles and predefined pathways between them on the physical terrain.

In the analytical studies for the performance analysis of wireless ad hoc networks, closed form expressions for the spatial node distribution are very desirable to understand long-run behavior of the network spatial behavior. For instance, the analysis that are presented in [11], [12], [13] to estimate the capacity per source to destination pair of these networks are significantly dependent on the spatial distribution of mobile nodes. Additionally, for some scenarios in which terminals can be highly mobile on a wide region, the spatial distribution of offered traffic may not be ignored in determining the capacity of asynchronous MAC layer protocols. Observe that the analysis of this case requires an accurate knowledge of the spatial distribution of nodes. The analytical work presented in [14] also considers the station locations for the MAC layer throughput analysis but the terminals are assumed to be uniformly distributed in the region, which may not be valid for different mobility scenarios. Moreover, this knowledge can be also used in evaluating the connectivity properties of ad hoc networks, which have been extensively studied in [15], [16]. In addition to these, the distribution of link distance between mobile terminals, which is an important characteristic of wireless ad hoc networks [17], [18], can be obtained from the spatial distribution of terminals.

Hence in this paper we propose a generalized random mobility model that is general enough to capture the major characteristics of a realistic movement profile, and yet is simple enough to mathematically formulate its long-run behavior with analytical expressions. The mobility pattern of a terminal that moves according to this generalized model is composed consecutive movement epochs in a closed region and it is uncorrelated with the movement behavior of other terminals. During each movement epoch, mobile terminal at first moves on the finite line segment joining the starting and destinations points of the epoch at a random speed and then it pauses at the destination for a random amount of time. The generality of our model is actually originating from the approach that we took to determine the destination point, movement speed, and pause time at the destination, and can be explained as follows:

- The distribution of the destination points are assumed to be general and can be conditionally dependent on the starting point of the movement epoch.
- The random speed for each epoch is drawn from a general distribution function that can be conditionally dependent on the starting and destination locations of the movement epoch, and the current location of mobile terminal if necessary.
- The pause time at each destination is selected randomly from a distribution that is dependent on the location of the destination point.

The fact that we make the mobility modeling with respect to these generalized approaches has number of advantages. First, since destinations are selected from a general distribution, a movement scenario in which terminals select some specific locations, for example, *hotspots*, as destination with higher probability, can be easily captured. Furthermore, some mobility scenarios may require a Markovian dependency between the destination points of consecutive movement epochs. For instance, the probability of selecting a hotspot as destination can be different from different starting points. This case can be naturally incorporated into our model by employing a distribution function for destinations that is conditionally dependent on the starting points.

Second, the generic approach for determining speed provides a unique opportunity to select speed according to the distance that is going to be traveled, and also a method to model variable speed during movement epochs. Clearly, if the speed of the terminal can vary during moving, then our model can even be used to capture different *acceleration* characteristics of vehicles. Finally, by employing a pause time distribution for each epoch that is a function of destination coordinate, we reached to the flexibility of pausing different times at at different locations.

For some sophisticated mobility models, performing its long-run analysis first over one-dimensional regions will be useful in gaining some insight into the methodology that has be followed for the analysis of higher dimensions. Thus, in this paper, we concentrate our analysis to one-dimensional regions, and develop an analytical framework that provide closed form expressions for the long-run location and speed distributions. We also believe that the analytical results presented can provide a methodology to analytically formulate the fundamental properties of wireless ad hoc networks for number sophisticated mobility scenarios (e.g., capacity, connectivity).

A. Related work

There have been a number of works attempting to obtain spatial node distribution for the ad hoc environments where terminals move according to random walk or random waypoint mobility models. The simulation studies that are presented in [19] and [20] for the random waypoint mobility model showed that the long-run spatial distribution of mobiles is independent from their initial placement in the simulation area, and also observed that resulting distribution is more accumulated at the center of the region. In [21], the movement pattern of the same mobility model is characterized as a stochastic process, and analytical expressions for the long-run location distribution are derived. In [22], authors not only concentrate on the analytical expressions for long-run spatial distribution of random waypoint model, but also on the limiting distribution of speed and procedures for the accurate simulation of this mobility model as well. The simulation study presented in [7] also concentrated in the same model, and examined average node speed at the steady-state. They pointed out that the closer v_{min} to zero, the more time it takes for the simulation of the mobility model to reach stability. In [8], this work is extended by analytical studies and authors provided steady-state average speed distribution for several random mobility models in which the speed for a movement epoch is chosen independently from the destination of that epoch. As a byproduct of their analytical formulation, authors also proposed a simulation methodology that eliminates the variations in the average nodal speed of these kinds of mobility models. In [23], authors provide an analytical framework for the steady-state speed and residual distance analysis of random waypoint like mobility models, and similar to [8], they also proposed methodology for the efficient simulation of those mobility models. In [24], a statistical analysis is done to identify the conditions in which the spatial node distribution of random waypoint mobility model, and a variant of two-dimensional random walk motion can be approximated with uniform distribution.

While each of the analytical and simulation studies mentioned above provide a comprehensive approach for the long-run characteristics of the random walk and waypoint like mobility models, none attempts to make major extensions on these models so that they describe a more realistic pattern. It is clear that, mobility models that are defined according to deterministic parameters such as predetermined pathways and obstacles, are more realistic than the random mobility models. However, as the deterministic dimension of the mobility model expands the possibility of deriving long-run properties of the model in terms of closed from expressions decreases. The most significant differences between the mobility model proposed in this paper and other random or deterministic models are the degrees of generality in mobility modeling and simplicity for the long-run analysis.

The next section provides the mobility formulation according to our mobility model, basic definitions, and our approach for long-run analysis. In the third section the analytical results are presented with example scenarios. Section IV concludes the paper.

II. Mobility Formulation

In this section, we provide the formal description of the generalized random mobility model introduced in Section I for one-dimensional mobility terrains, and construct an analytical framework for its long-run analysis. Let $R = [0, a]$ represent the region on which mobile terminals operate, and denote $X_s \in R$ and $X_d \in R$ as the random variables corresponding to the starting and destination points of a movement epoch, respectively. Furthermore, let the random variable V defined on the state space $[v_{\min}, v_{max}]$, where $v_{min} > 0$, denote the speed of a terminal while moving from X_s to X_d. In addition, denote the random variable T_p with state space

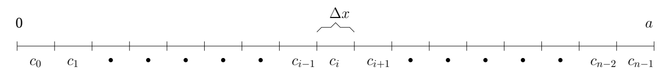

Fig. 1. Discretization of $R = [0, a]$ according to cells of size $\Delta x = \frac{a}{n}$.

$[0, \infty)$ as the pause time spent at destination point X_d. With respect to these notations, and the mobility modeling approach we proposed in this paper, we define the following parameters:

- $f_{X_d|X_s}$: the conditional probability density function (pdf) of X_d given X_s,
- $f_{V|X_s,X_d}$: the conditional pdf of V given X_s and X_d,
- $f_{T_p|X_d}$: the conditional pdf of T_p given X_d.

Hence, the mobility formulation that is performed according to the generalized random mobility model can be characterized by the triplet $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$.

Before we proceed further, we note that X_s and X_d actually represent the destination points of any two consecutive movement epochs, and the conditional pdf $f_{X_d|X_s}$ that identifies the distribution of X_d given X_s at the embedded points in time where a new epoch starts, is referred as *stochastic density kernel* by Feller [25]. We will identify the restrictions on the choice of $f_{X_d|X_s}$ required for the long-run characterization as we proceed further in the analysis.

Now as we have noted in Section I, each terminals movement is assumed to be independent from others. Thus, it is enough to model a single terminals behavior for the long-run analysis. For this purpose, let $\mathbf{X}(t)$ denote the state of the mobile terminal at time t. According to the specifications of the mobility model we proposed, the stochastic process $\{\mathbf{X}(t), t \geq 0\}$ must be defined on a state space that has separate dimensions for current location, destination, and speed, and more importantly, the ranges of these dimensions must be continuous. However, in the analytical framework we construct, we use a discretization method and describe the mobility behavior of nodes with a stochastic process that is defined on a multidimensional *discrete* state space. In addition, instead of observing the state of a terminal continuously, we will observe it at embedded times T_k, for $k \in \mathbf{N}$, such that $T_0 = 0,\ T_{k+1} \geq T_k,\ \forall k \in \mathbf{Z}^+$. Also, these embedded times are dependent on the evolution of the system that dictates the movement behavior of the mobile node. The following list formally defines the assumptions that the analytical framework is built on:

- A_1: The region R is discretisized into n cells of the same size, that are denoted by $c_i = [(i-1)\Delta x, i\Delta x]$, $i = 0 \ldots n-1$, as shown in Fig. 1, where $\Delta x = \frac{a}{n}$ for $n > 1$. A mobile terminal is assumed to occupy one of the c_i's at any moment in time, and movement epochs start from a cell and ends up at a different destination cell.
- A_2: The random variable V, which denotes the speed of a mobile during a movement epoch, is approximated by the discrete random variable V^* taking values in the state space
$$\mathcal{S}_{V^*} = \{z_1, z_2, \ldots, z_m\}, \tag{1}$$
where $z_r = r\,\Delta v$, $r = 1, \ldots, m$, for some discretization parameter $\Delta v > 0$, and $m > 1$ such that $\Delta v \leq v_{\min}$ and

$v_{\max} \leq m\Delta v$.

A_3: Observation time T_k point to the time of occurrence of one of the following events:

- E_1: The terminal, which is in pause mode, selects a new destination that is different from the current cell occupied, and jumps into moving state at the current cell,
- E_2: The terminal, which is traveling in the direction of the target cell, moves out from the current cell and enters the neighbor cell that lies on the path between the current and destination cells,
- E_3: The terminal reaches to the destination cell and enters the pause mode at that location.

Notice that the higher the degree of discretization for the closed region R is selected, the better approximation can be done to the exact location of the terminals. Also, as the discretization parameter $\Delta v \rightarrow 0$ (i.e., $m \rightarrow \infty$), the discrete approximating random variable V^* becomes indistinguishable from the original random variable V. Therefore, as $\{n, m\} \rightarrow \infty$, we converge to model with continuous state space. For the rest of this paper, we will use the term *discretisized mobility formulation* to refer to the version of the generalized random mobility modeling approach that is constructed according to the assumptions A_1, A_2, and A_3.

Now, let $\mathbf{S}_k$, $k \in \mathbf{N}$, denote the state of the mobile terminal at time T_k. Given the assumptions A_1, A_2 and A_3, the finite-state space of $\mathbf{S}_k$ will be defined as follows:

$$\begin{aligned} \mathcal{S} &= \{(c_i, c_j, z_r, q) \mid i, j = 0, \ldots, n-1, i \neq j, \\ &\quad\; r = 1, \ldots, m, q = 1\} \\ &\cup \;\; \{(c_i, q) \mid i = 0, \ldots, n-1, q = 0\} \end{aligned} \tag{2}$$

where c_i is the current cell occupied, c_j is the destination cell, z_r is the discretisized speed, and q is the indicator of being in the mode of moving towards the target cell, or pausing at the destination.

Hence, the stochastic process $\{\mathbf{X}(t), t \geq 0\}$ that represents the state of the mobile terminal at time t, can be redefined on the finite-state space $\mathcal{S}$ by the following expression:

$$\mathbf{X}(t) = \mathbf{S}_k, \quad \text{if} \;\; T_k \leq t < T_{k+1}$$

where the times $T_1, T_2, \ldots$ are the successive times of transitions of $\mathbf{X}(t)$, and $\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2, \ldots$ represent the successive states occupied by $\mathbf{X}(t)$.

Observe that by constructing a state space that has a separate dimension for the destination cell of moving terminals, the future evolution of the stochastic process $\{\mathbf{S}_k, k \in \mathbf{N}\}$ becomes dependent only on the current state of the mobile terminal, not on its history at previous observation points. Furthermore, assume that current state occupied by $\mathbf{X}(t)$ is s. Once the state $s' \in \mathcal{S}$ has been selected with some probability as the next state to be visited, the distribution of sojourn time in state s can be determined from the components of state s. Consequently, the following relationship will be valid for all $k \in \mathbf{N}$, and all possible sets $\{s, s'\} \subset \mathcal{S}$.

$$\begin{aligned} &\Pr\{\mathbf{S}_{k+1} = s', T_{k+1} - T_k \leq t | \mathbf{S}_k = s, T_k, \ldots, \mathbf{S}_0, T_0\} \\ &= \Pr\{\mathbf{S}_{k+1} = s', T_{k+1} - T_k \leq t \,|\, \mathbf{S}_k = s\} \end{aligned}$$

TABLE I

TRANSITION PROBABILITIES OF THE PROCESS $\{\mathbf{S}_k, k \in \mathbf{N}\}$

Event	Transition	Probability	Condition*
E_1	$(c_i, 0) \rightarrow (c_i, c_j, z_r, 1)$	$\frac{\tau_{j\|i}}{1-\tau_{i\|i}} \nu_{r\|i,j}$	$i \neq j$
E_2	$(c_i, c_j, z_r, 1) \rightarrow (c_{i+1}, c_j, z_r, 1)$	1	$j > i+1$
	$(c_i, c_j, z_r, 1) \rightarrow (c_{i-1}, c_j, z_r, 1)$	1	$j < i-1$
E_3	$(c_i, c_j, z_r, 1) \rightarrow (c_j, 0)$	1	$\|i - j\| = 1$

* $i, j = 0, \ldots, n-1$, $r = 1 \ldots m$

Therefore, the stochastic process $\{\mathbf{S}_k, T_k; k \in \mathbf{N}\}$ with finite-state space $\mathcal{S}$ satisfies the conditions for being *Markov Renewal Process*, and the process $\{\mathbf{X}(t), t \geq 0\}$ can be called as the *semi-Markov process* (SMP) associated with $\{\mathbf{S}_k, T_k; k \in \mathbf{N}\}$ [26]. Moreover, since the general distributions for destination, speed, and pause time parameters are assumed to be *time-homogeneous* in the model proposed, for each pair $(s, s') \in \mathcal{S} \times \mathcal{S}$, the distribution of state holding time in state s before moving to state s', given that the next state to be visited is s', would be independent of k. Hence, based on the results provided in [26] and [27] for the theory of semi-Markov processes, the transitions of the process $\mathbf{X}(t)$ from state s to state s' at the time instants T_k can be governed by the *discrete-time* Markov chain (DTMC) $\{\mathbf{S}_k, k \in \mathbf{N}\}$ with finite-state space $\mathcal{S}$ and transition probability matrix $P = [p_{s\,s'}]$, where $p_{s\,s'} = \Pr\{\mathbf{S}_{k+1} = s' \,|\, \mathbf{S}_k = s\}$, such that $\sum_{s' \in \mathcal{S}} p_{s\,s'} = 1$ for all $s \in \mathcal{S}$. The process $\{\mathbf{S}_k, k \in \mathbf{N}\}$ is also called *embedded* DTMC of SMP.

Consequently, if the DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$ satisfies the *ergodicity* conditions, and if the mean state holding times are finite, then the SMP $\{\mathbf{X}(t), t \geq 0\}$ can be characterized at the long-run. Clearly, if long-run proportion of times spent at the states of the discrete state space $\mathcal{S}$ are known, then by aggregating the states that has the same current cell component, that is, c_i, the long-run location distribution for the discretisized region can be easily obtained. After this, by observing the limiting behavior of that discrete result as $n \rightarrow \infty$ and $m \rightarrow \infty$, the continuous result can be derived. The same approach can be also used to obtain long-run speed distribution but in that case, the states with the same speed component, that is, z_r, must be aggregated. In the following section, we will at first generate the irreducible stochastic matrix P explicitly. Then, we will apply this approach to derive long-run location and speed distributions of continuous case.

III. Analytical Results for Discretisized and Continuous Mobility Formulations

In this section, we apply our solution framework with the ultimate aim of finding closed form expressions for the long-run location and speed distributions over the given one-dimensional mobility terrain $R = [0, a]$.

Now to describe the transition probabilities of the embedded DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$, we first define:

$$\begin{aligned} \tau_{j|i} &= \Pr\{X_d \in c_j | X_s \in c_i\} \\ &= \int_{j\,\Delta x}^{(j+1)\,\Delta x} dx_d \, f_{X_d|X_s}(x_d | X_s \in c_i), \end{aligned} \tag{3}$$

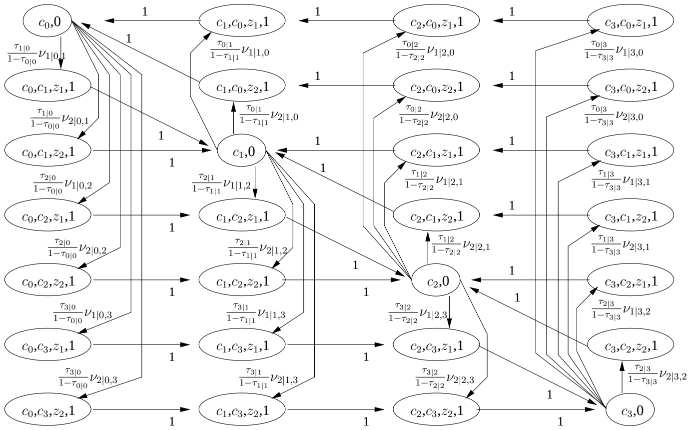

Fig. 2. State transition diagram for the process $\{\mathbf{S}_k, k \in \mathbf{N}\}$, where $n = 4$ and $m = 2$.

for $i, j = 0, \ldots, n-1$. Next, since V is allowed to be dependent on X_s and X_d, we define the probability mass function of V^* given $X_s \in c_i$ and $X_d \in c_j$, that is, for a movement epoch that had started at c_i and destined to c_j, by

$$\begin{aligned}\nu_{r|i,j} &= \Pr\{V^* = z_r | X_s \in c_i, X_d \in c_j\} \\ &= \int_{(r-1)\Delta v}^{r\Delta v} f_{V|X_s,X_d}(v | X_s \in c_i, X_d \in c_j)\, dv, \end{aligned} \quad (4)$$

for $r = 1, \ldots, m$.

Based on the events E_1, E_2, and E_3 that cause state changes, and $\tau_{j|i}$ and $\nu_{r|i,j}$ and given above, the possible transitions and the corresponding transition probabilities of the embedded DTMC can be grouped as in Table I.

It should be noted from Table I that when E_1 occurs, the mobile that is located at c_i jumps to moving mode in the current cell occupied. We enforced these transitions for the purpose of uniquely identifying moving and pausing terminals. In Fig. 2 we depicted the state transition diagram of the process $\{\mathbf{S}_k, k \in \mathbf{N}\}$ for a simple case where n=4 and m=2.

Next, we formulate the transition probability matrix P of the process $\{\mathbf{S}_k, k \in \mathbf{N}\}$ in full generality. Clearly the structure of the matrix P depends on the order imposed on the states in $\mathcal{S}$. The ordering that we have decided on is $\mathcal{S} = \{\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{n-1}\}$, where each $\mathcal{S}_i$ has $m(n-1)+1$ states according to the following order:

$$\begin{aligned}\mathcal{S}_i = \{&(c_i, c_0, z_1, 1), \ldots, (c_i, c_0, z_m, 1), \ldots, \\ &(c_i, c_{i-1}, z_1, 1), \ldots, (c_i, c_{i-1}, z_m, 1), (c_i, 0), \\ &(c_i, c_{i+1}, z_1, 1), \ldots, (c_i, c_{i+1}, z_m, 1), \ldots, \\ &(c_i, c_{n-1}, z_1, 1), \ldots, (c_i, c_{n-1}, z_m, 1)\}. \end{aligned} \quad (5)$$

Based on this ordering, the transition probability matrix P has the following discrete-time level-dependent quasi-birth-and-death process (QBD) form [28]:

$$P = \begin{bmatrix} A_1^{(0)} & A_0^{(0)} & & & \\ A_2^{(1)} & A_1^{(1)} & A_0^{(1)} & & \\ & \ddots & \ddots & \ddots & \\ & & A_2^{(n-2)} & A_1^{(n-2)} & A_0^{(n-2)} \\ & & & A_2^{(n-1)} & A_1^{(n-1)} \end{bmatrix} \quad (6)$$

where the matrices $A_0^{(i)}$, $A_1^{(i)}$, and $A_2^{(i)}$, $i = 0, \ldots, n-1$, are $(m(n-1)+1) \times (m(n-1)+1)$, and defined as

$$A_0^{(i)} = \left[\begin{array}{c|c|c} & \boldsymbol{e}_0^{(i)} & \\ \hline & I_1 & \\ \hline & & I_{m(n-i-2)} \end{array}\right], \quad A_2^{(i)} = \left[\begin{array}{c|c|c} I_{m(i-1)} & & \\ \hline & I_1 & \\ \hline & \boldsymbol{e}_2^{(i)} & \end{array}\right],$$

$$A_1^{(i)} = \left[\begin{array}{ccc|c|ccc} & & & & & & \\ \hline B_0^{(i)} & \ldots & B_{i-1}^{(i)} & & B_{i+1}^{(i)} & \ldots & B_{n-1}^{(i)} \\ \hline & & & & & & \end{array}\right] \quad (7)$$

where upper left block of $A_1^{(i)}$ is a zero matrix of size $m\,i \times m\,i$, and I_h denote the $h \times h$ identity matrix for some positive integer h. Moreover, the $1 \times m$ row vector $B_j^{(i)}$, for $i, j = 0, \ldots, n-1$, where $i \neq j$, and column vectors $\boldsymbol{e}_0^{(i)}$, $\boldsymbol{e}_2^{(i)}$ of respective sizes $m(i+1) \times 1$ and $m(n-i) \times 1$, are respectively defined by

$$\begin{aligned} B_j^{(i)} = \frac{\tau_j}{1-\tau_i}\boldsymbol{\nu}_{m|i,j}, \quad \boldsymbol{e}_0^{(i)} &= [0, \ldots, 0, e_{m-1}]^{\mathrm{T}}, \\ \boldsymbol{e}_2^{(i)} &= [e_{m-1}, 0, \ldots, 0]^{\mathrm{T}} \end{aligned} \quad (8)$$

where $\boldsymbol{\nu}_{m|i,j} = [\nu_{1|i,j}, \ldots, \nu_{m|i,j}]$, and e_h is the $1 \times h$ vector of ones. The remaining blocks of the matrices $A_0^{(i)}$, $A_1^{(i)}$, and $A_2^{(i)}$ are zero matrices of sizes that can be easily derived from the dimensions of the other blocks.

Before we can proceed with the long-run analysis of the SMP $\{\mathbf{X}(t), t \geq 0\}$, we must first find the steady-state distribution of the embedded DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$ with the transition probability matrix P given in (6). Clearly, this distribution exists if and only if a steady-state distribution exists for X_s [1], and $\{\mathbf{S}_k, k \in \mathbf{N}\}$ satisfies the ergodicity conditions. Hence, we focus on these issues now.

Under the "mild" regularity conditions defined by Feller [25] on $f_{X_d|X_s}(x_d|x_s)$, there exists a steady-state distribution for X_s with pdf $f_{X_s}(x_d)$, which can be uniquely determined from the solution of the following integral equation

$$f_{X_s}(x_d) = \int_0^a f_{X_d|X_s}(x_d|x_s) f_{X_s}(x_s) dx_s \quad (9)$$

We note that the integral equation given above, which is used to obtain the steady-state behavior of the discrete-time [2], continuous-state Markov process $\{X_s\}$, has an analogy to the linear system $\varphi T = \varphi$, with $\|\varphi\|_1 = 1$ where $T = [\tau_{j|i}]$. Basically, it is the *analog* version of $\varphi T = \varphi$. Clearly if the distribution of X_d is assumed to be independent from X_s, then the solution of the integral equation (9) would be simple. However, for other cases, deriving $f_{X_s}(x_d)$ can be a very tedious task. We will return back this point later in Subsection III-B that concentrates on the mobility scenarios where the choice of X_d is dependent on X_s.

Hence, if the pdf $f_{X_s}(x_d)$ can be uniquely determined from the solution of (9), then the probability of starting a movement

[1]Since X_d is the X_s of the next mobility epoch, X_s and X_d can be used interchangeably at the long-run.

[2]The stochastic process $\{X_s\}$ changes its state at embedded time instants that represent the starting time of a new movement epoch.

epoch from cell c_i at the steady-state, which is denoted by φ_i, $i = 0, \ldots, n-1$, will be given by

$$\varphi_i = \int_{i\,\Delta x}^{(i+1)\,\Delta x} dx_d\, f_{X_s}(x_d) \tag{10}$$

Next, we examine the ergodicity of $\{\mathbf{S}_k, k \in \mathbf{N}\}$.

Lemma 1: If the pdf $f_{X_s}(x_d)$ can be uniquely determined from the integral equation (9), and if $\nu_{r|i,j} > 0$, $i, j = 0, \ldots, n-1$ and $r = 1, \ldots, m$, then the embedded DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$ defined on state space $\mathcal{S} = \{\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_{n-1}\}$, with transition probability matrix P defined as in (6), will be irreducible and aperiodic.

Proof: Please refer to Appendix. ■

Thus, when the conditions of *ergodicity* for the DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$ are satisfied, the steady-state distribution of it, which we denote by π_s for state $s \in \mathcal{S}_i$, $i = 0 \ldots n-1$, can be uniquely determined by solving the matrix equation

$$\boldsymbol{\pi} P = \boldsymbol{\pi}, \text{ with } \|\boldsymbol{\pi}\|_1 = 1 \tag{11}$$

where $\boldsymbol{\pi} = [\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_{n-1}]$, and $\boldsymbol{\pi}_i$ is a (row) vector of size $m(n-1)+1$ whose elements are π_s, $\forall s \in \mathcal{S}_i$, according to the order given by (5). $\boldsymbol{\pi}_i$ can be also called the solution vector for level i, $i = 0, \ldots, n-1$, as in [29].

Next, we examine the solution of the linear system given by (11). To the best of our knowledge, if there are no additional assumptions made on the properties of the matrix P, the most efficient direct computational procedure to find the steady-state distribution of finite-state level-dependent QBDs is presented in [28]. By using that procedure, one can obtain $\boldsymbol{\pi}$ numerically for some moderate values of n and m. However, as we made clear before, we are aimed at finding the limiting behavior of the long-run distributions for the discretisized case as $\{n, m\} \to \infty$. Clearly this can only be done after deriving the location and speed distributions in closed from expressions. Therefore, we focused on an alternative direct approach and derived the following result.

Lemma 2: If the conditions given in Lemma 1 for the ergodicity of the DTMC $\{\mathbf{S}_k, k \in \mathbf{N}\}$ are satisfied, then the solution vector $\boldsymbol{\pi}_i$ for level i, $i = 0, \ldots, n-1$, of the level-dependent QBD process given in (6), with the matrices $A_0^{(i)}$, $A_2^{(i)}$, and $A_1^{(i)}$ defined as in (7), is given by

$$\boldsymbol{\pi}_i = [\boldsymbol{\pi}_{i,0}, \ldots, \boldsymbol{\pi}_{i,i}, \ldots, \boldsymbol{\pi}_{i,n-1}]/N \tag{12}$$

where

$$\boldsymbol{\pi}_{i,j} = \begin{cases} \sum\limits_{\ell=i}^{n-1} \varphi_\ell\, \tau_{j|\ell}\, \boldsymbol{\nu}_{m|\ell,j}, & \text{if } j < i \\ \varphi_i\, (1-\tau_{i|i}), & \text{if } j = i \\ \sum\limits_{\ell=0}^{i} \varphi_\ell\, \tau_{j|\ell}\, \boldsymbol{\nu}_{m|\ell,j}, & \text{if } j > i \end{cases}, \tag{13}$$

and $N = \sum_{i=0}^{n-1} \|\boldsymbol{\pi}_i\|_1$.

Proof: Please refer to Appendix. ■

To characterize the SMP $\{\mathbf{X}(t), t \geq 0\}$ at the long-run, it remains to formulate the expected state holding times. For this purpose, let $\bar{t}_s$ be the expected holding time in state $s \in \mathcal{S}$. Recall that in Section II, we decomposed the state space $\mathcal{S}$ into two groups that represent moving (i.e., $q = 1$), and pausing (i.e., $q = 0$) terminals. Therefore, since $R = [0, a]$ is discretisized by cells of size Δx, the expected time that is going to be spent in a cell c_i by moving terminals is simply

$$\bar{t}_s = \frac{\Delta x}{z_r} \tag{14}$$

where $s = (c_i, c_j, z_r, 1)$, $i, j = 0, \ldots, n-1$, and $r = 1, \ldots, m$, such that $i \neq j$. To formulate the mean time that is spent in a state of the form $s = (c_i, 0)$, $i = 0, \ldots, n-1$, we also define the following notation:

$$\begin{aligned} \bar{t}_s = E[T_{p_i}] &= E[T_p | X_s \in c_i] \\ &= \int_0^\infty \Pr\{T_p > t_p | X_s \in c_i\}\, dt_p \end{aligned} \tag{15}$$

Notice that the following equation

$$\sum_{s \in \mathcal{S}} \pi_s \bar{t}_s < \infty \tag{16}$$

is satisfied only if the minimum speed a mobile can attain is nonzero, and mean pause time spent at destinations are finite. Hence, if the mobility characterization parameters f_V and $f_{T_p|X_d}$ are selected appropriately to satisfy these conditions, then the conditions given in [26] for the long-run characterization of SMPs are satisfied, and P_s, which corresponds to long-run proportion of time that the process is in state s, is simply

$$P_s = \frac{\pi_s \bar{t}_s}{\sum\limits_{s' \in \mathcal{S}} \pi_{s'} \bar{t}_{s'}}, \quad \forall s \in \mathcal{S} \tag{17}$$

Finally, after aggregating the states that belong to the same level (i.e., $\mathcal{S}_i$, $i = 0, \ldots, n-1$) of the level-dependent QBD process $\{\mathbf{S}_k, k \in \mathbf{N}\}$, we obtained the following result for the long-run location distribution of the discretisized one-dimensional regions.

Lemma 3: For the mobile terminal, whose mobility pattern is formulated according to the discretisized version of the $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$ mobility characterization, let p_i, $i = 0, \ldots, n-1$, denote the long-run proportion of time that terminal stays in cell c_i. If the conditions given in Lemma 1 holds, and if the equation (16) is satisfied, then

$$p_i = \frac{\varphi_i\,(1-\tau_{i|i}) E[T_{p_i}] + k_i\, \Delta x}{\sum\limits_{\imath=0}^{n-1} \varphi_\imath\,(1-\tau_{\imath|\imath}) E[T_{p_\imath}] + \hat{D}_n\, \Delta x} \tag{18}$$

where

$$\begin{aligned} k_i &= \sum_{j=0}^{i-1} \sum_{\ell=i}^{n-1} \varphi_\ell\, \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j} \\ &+ \sum_{j=i+1}^{n-1} \sum_{\ell=0}^{i} \varphi_\ell\, \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j}, \end{aligned} \tag{19}$$

and

$$\hat{D}_n = \sum_{\imath=0}^{n-1} k_\imath \tag{20}$$

Proof: Please refer to Appendix. ■

Next, we turn our attention to the limiting behavior of the discrete result derived in Lemma 3, and summarize our fundamental result for the long-run location distribution.

Theorem 1: For the mobile terminal, whose mobility pattern is characterized by $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$, let $f_X(x)$, $x \in [0,a]$, denote the pdf of its location distribution at the long-run. If the pdf $f_{X_s}(x_d)$ can be uniquely determined from the integral equation (9), and $E[T_p|X_s = x_s] < \infty$, $\forall x_s \in [0,a]$, and $f_{V|X_s,X_d} > 0$, $\forall v \in [v_{\min}, v_{\max}]$, and $\forall x_s, x_d \in [0,a]$, then

$$f_X(x) = \frac{f_{X_s}(x)E[T_p|X_s = x] + k_X(x)}{E[T_p|0 \le X_s \le a] + \hat{D}} \tag{21}$$

where

$$k_X(x) = \int_0^x dx_d \int_x^a dx_s\, g_X(x_s,x_d) + \int_x^a dx_d \int_0^x dx_s\, g_X(x_s,x_d) \tag{22}$$

where

$$g_X(x_s,x_d) = f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) E[\tfrac{1}{V}|X_s=x_s,X_d=x_d], \tag{23}$$

and

$$E[\tfrac{1}{V}|X_s=x_s,X_d=x_d] = \int_{v_{\min}}^{v_{\max}} dv\, \frac{1}{v}\, f_{V|X_s,X_d}(v|x_s,x_d), \tag{24}$$

and

$$\hat{D} = \int_0^a dx\, k_X(x) \tag{25}$$

Proof: Please refer to Appendix. ■

It should be noted that if the distribution of V is independent from X_s and X_d, then the pdf f_V can be employed instead of $f_{V|X_s,X_d}$ for mobility characterization, and k_X (22), and $\hat{D}$ (25) simplifies to

$$\begin{aligned} k_X(x) &= E[\frac{1}{V}] \int_0^x dx_d \int_x^a dx_s\ f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \\ &+ E[\frac{1}{V}] \int_x^a dx_d \int_0^x dx_s\ f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \end{aligned} \tag{26}$$

$$\hat{D} = E[\frac{1}{V}] \bar{D} \tag{27}$$

where

$$\begin{aligned} \bar{D} &= \int_0^a dx_d \int_0^{x_d} dx_s (x_d - x_s) f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \\ &+ \int_0^a dx_d \int_{x_d}^a dx_s (x_s - x_d) f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \end{aligned} \tag{28}$$

Notice that $\bar{D}$ is actually the average distance between the two points X_s and X_d drawn at random according to the pdfs f_{X_s}, and $f_{X_d|X_s}$, respectively. In addition, if X_d is also independent from X_s, then f_{X_d} can be used instead of $f_{X_d|X_s}$, and k_X (22) further simplifies to

$$k_X(x) = 2\ E[\frac{1}{V}] \int_0^x dx_d \int_x^a dx_s\ f_{X_d}(x_s) f_{X_d}(x_d) \tag{29}$$

Having defined the long-run location distribution, we now concentrate on the long-run speed distribution. Clearly in order to achieve this, we need to aggregate the steady-state probabilities of the states in $\mathcal{S}$ that has the same speed component, and take the limit of the resulting expression as $\{\Delta x, \Delta v\} \to 0$. Thus, for the mobile terminal whose movement behavior is characterized according to the triplet $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$, let the continuous random variable $\tilde{V}(t)$ defined on the state space $\{0\} \cup \{v | v_{min} \le v \le v_{max}\}$ denote the speed of a mobile terminal at time t. Note that, since the mobile can be in pausing mode at some point in time, $\tilde{V}(t)$ can also attain the zero value. Next, let $\tilde{V}$ represent the random variable having the long-run distribution of $V(t)$, and denote its pdf by $f_{\tilde{V}}$. Finally, referring back to assumption A_2, denote the discrete approximation to the continuous random variable $\tilde{V}$ by $\tilde{V}^*$. Clearly, the state space of $\tilde{V}^*$ must be

$$\mathcal{S}_{\tilde{V}^*} = \{0\} \cup \mathcal{S}_{V^*} = \{z_0, z_1, z_2, \ldots, z_m\} \tag{30}$$

where $z_r = r\Delta v$, $r = 0,1,\ldots,m$.

Now, let ψ_r denote the long-run proportion of time that a mobile possesses speed z_r, $r = 0,1,\ldots,m$. After aggregating the components of the vectors $\boldsymbol{\pi}_{i,j}$, $i,j = 0,\ldots,n-1$, defined by (13) according to the states in $\mathcal{S}$ that have the same speed component, and using the mean times that are going to be spent in those states we get

$$\psi_r = \begin{cases} \Big(\sum_{i=0}^{n-1} \varphi_i\,(1-\tau_{i|i}) E[T_{p_i}]\Big)/\hat{N}, & \text{if } r = 0 \\ \Big(\sum_{i=0}^{n-1} \Delta x \Big(\sum_{j=0}^{i-1}\sum_{\ell=i}^{n-1} \varphi_\ell\ \tau_{j|\ell}\ \frac{1}{z_r}\nu_{r|\ell,j} \\ \quad + \sum_{j=i+1}^{n-1}\sum_{\ell=0}^{i} \varphi_\ell\ \tau_{j|\ell}\ \frac{1}{z_r}\ \nu_{r|\ell,j}\Big)\Big)/\hat{N}, & \text{else} \end{cases} \tag{31}$$

where $\hat{N} = \sum_{\imath=0}^{n-1} \varphi_\imath\,(1-\tau_{\imath|\imath}) E[T_{p_\imath}] + \hat{D}_n\ \Delta x$. Taking the limit of this discrete result as $\{\Delta x, \Delta v\} \to 0$, we reached to the following theorem, which we state without proof.

Theorem 2: For the mobile terminal, whose mobility pattern is characterized by $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$, if the conditions that are given in Theorem 1 for the parameters $f_{X_d|X_s}$, $E[T_p|X_s = x_s]$, and $f_{V|X_s,X_d}$ are satisfied, then

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p|0\le X_s\le a]\delta(\tilde{v})}{E[T_p|0\le X_s\le a]+\hat{D}}, & \text{if } \tilde{v} = 0 \\ \frac{\int_0^a dx\, k_{\tilde{V}}(x,\tilde{v})}{E[T_p|0\le X_s\le a]+\hat{D}}, & \text{if } \tilde{v} \in [v_{\min}, v_{\max}] \end{cases}, \tag{32}$$

and

$$E[\tilde{V}] = \frac{\bar{D}}{E[T_p|0 \le X_s \le a] + \hat{D}} \tag{33}$$

where

$$\begin{aligned} k_{\tilde{V}}(x,\tilde{v}) &= \int_0^x dx_d \int_x^a dx_s g_{\tilde{V}}(x_s,x_d,\tilde{v}) \\ &+ \int_x^a dx_d \int_0^x dx_s g_{\tilde{V}}(x_s,x_d,\tilde{v}), \end{aligned} \tag{34}$$

where

$$g_{\tilde{V}}(x_s,x_d,\tilde{v}) = f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \frac{1}{\tilde{v}} f_{V|X_s,X_d}(\tilde{v}|x_s,x_d) \tag{35}$$

It should be noted that, if the distributions of X_d and T_p are independent from X_s, and if distribution of V is also independent of X_s and X_d, then the mobility characterization can be done by the triplet $< f_{X_d}, f_V, f_{T_p} >$, and the formulation of $f_{\tilde{V}}(v)$ and $E[\tilde{V}]$ for this simplified mobility formulation will match to the results that are derived in [8] for a class of mobility models where speed is selected independently from the distance that is going to be traveled.

Finally, from the results presented by Theorems 1 and 2, it is clear that the dependency of X_d on X_s makes the fundamental difference. Therefore, in the following two subsections, we will at fist concentrate on some example scenarios that uses f_{X_d} (i.e. distribution of X_d is independent from X_s) for mobility characterization. Then, we will proceed to more complicated scenarios by employing the stochastic density kernel $f_{X_d|X_s}$ for mobility formulation.

A. Variants of mobility characterizations done by f_{X_d}

Example 1: The random waypoint model [3] represents the simplest nontrivial case of our generalized modeling approach, and can be characterized according to the triplet $< f_{X_d}, f_V, f_{T_p|X_d} >$, where the parameters are defined by

$$f_{X_d}(x_d) = \begin{cases} \frac{1}{a}, & \text{if } 0 \leq x_d \leq a \\ 0, & \text{otherwise} \end{cases}, \tag{36}$$

$$f_V(v) = \begin{cases} \frac{1}{v_{\max} - v_{\min}}, & \text{if } v_{\min} \leq v \leq v_{\max} \\ 0, & \text{otherwise} \end{cases} \tag{37}$$

and

$$f_{T_p|X_d}(t_p|x_d) = \begin{cases} h(t_p), & \text{if } t_p \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

where $h(t_p)$ is the pdf of the random variable T_p, which is independent from the location of the destination. Denoting the average time spent at the destinations by $E[T_p]$ (i.e., $E[T_p] = \int_0^\infty t_p\, h(t_p)\, dt_p$), observing

$$E[\frac{1}{V}] = \frac{\ln(\frac{v_{\max}}{v_{\min}})}{(v_{\max} - v_{\min})}, \tag{39}$$

and using Theorems 1 and 2, we obtained the following for the pdf of the long-run location distribution and the expected value of speed at the long-run:

$$f_X(x) = \frac{\frac{1}{a}E[T_p] + \frac{2x(a-x)}{a^2}E[\frac{1}{V}]}{E[T_p] + \frac{a}{3}E[\frac{1}{V}]}, \tag{40}$$

$$E[\tilde{V}] = \frac{a/3}{E[T_p] + \frac{a}{3}E[\frac{1}{V}]} \tag{41}$$

We note that if the speed choice for each movement epoch is deterministic with a parameter v, then $E[\frac{1}{V}]$ must be substituted with $\frac{1}{v}$. In addition, the analytical work presented in [21], considers two different limited variations of the one-dimensional result we derived for location distributions. At first, they concentrate on the case where $E[T_p] = 0$. Next, they extend their analysis, and provide the location distribution for the scenario where pause time is nonzero, and speed is deterministic (i.e., constant speed). For these two cases, if we make the appropriate changes in the formation of f_X given in (40) (i.e., $E[T_p] = 0$ for first case, and $E[\frac{1}{V}] = \frac{1}{v}$ for the other case), then the results will match the pdfs presented in [21].

Example 2: In the random waypoint mobility model we analyzed by Example 1, V is assumed to be independent from $|X_s - X_d|$, that is, the distance traveled during a movement epoch. However, in most of the realistic scenarios, V tends to increase as $|X_s - X_d|$ does. Thus, for this example, we make an improvement on the random waypoint model by proposing a $f_{V|X_s,X_d}$ that provides the opportunity to determine V proportional to the random variable $D = |X_s - X_d|$ with high probability.

Now, considered a truncated normal distribution [30] for V according to the pdf given by

$$f_{V|X_s,X_d}(v|x_s,x_d) = \frac{Z(\frac{v-\mu(x_s,x_d)}{\sigma})}{\sigma\big(\Phi(\frac{v_{\max}-\mu(x_s,x_d)}{\sigma}) - \Phi(\frac{v_{\min}-\mu(x_s,x_d)}{\sigma})\big)} \tag{42}$$

for $v_{\min} \leq v \leq v_{\max}$ where $\sigma > 0$, and

$$\mu(x_s,x_d) = v_{\min} + \frac{(v_{\max} - v_{\min})}{a}|x_s - x_d| \tag{43}$$

Z and Φ are the probability density and cumulative distribution functions for the normal distribution [30].

Hence, we reached to the following results for this improved case:

$$f_X(x) = \frac{\frac{1}{a}E[T_p] + k_X(x)}{E[T_p] + \hat{D}}, \tag{44}$$

$$E[\tilde{V}] = \frac{a/3}{E[T_p] + \hat{D}} \tag{45}$$

where

$$k_X(x) = \frac{2}{a^2}\int_0^x dx_d \int_x^a dx_s \int_{v_{\min}}^{v_{\max}} dv \frac{1}{v} f_{V|X_s,X_d}(v|x_s,x_d) \tag{46}$$

where $f_{V|X_s,X_d}$ is defined by (42).

Clearly, because of the complicatedness of $f_{V|X_s,X_d}$, $k_X(x)$ can only be evaluated numerically for a given $x \in [0,a]$, and also $\hat{D}$. However, for the extreme case $\sigma \to 0$, we have

$$f_{V|X_s,X_d}(v|x_s,x_d) = \delta(v - \mu(x_s,x_d)) \tag{47}$$

From a different point of view, for the limiting case where $\sigma \to 0$, V will be linearly dependent to $|X_s - X_d|$ with respect to the following transformation:

$$V = v_{\min} + \frac{(v_{\max} - v_{\min})}{a}|X_s - X_d| \tag{48}$$

Thus, the $k_X(x)$ given by (46) simplifies to

$$\begin{aligned} k_X(x) = 2\Big(& \ln((v_{\max}(a-x)+xv_{\min})/a)(x(v_{\max}-v_{\min})-v_{\max}a) \\ &+ \ln((x(v_{\max}-v_{\min})+v_{\min}a)/a)(x(v_{\min}-v_{\max})-av_{\min}) \\ &+ av_{\min}\ln(v_{\min})+av_{\max}\ln(v_{\max})\Big)/\Big(a(v_{\max}-v_{\min})^2\Big), \end{aligned} \tag{49}$$

and $\hat{D}$ will be given by

$$\hat{D} = \frac{a(v_{\max}^2 - v_{\min}^2 - 2v_{\min}v_{\max}\ln(\frac{v_{\max}}{v_{\min}}))}{(v_{\max} - v_{\min})^3} \tag{50}$$

Now, after substituting the $\hat{D}$ given above by (50) to the equation for $E[\tilde{V}]$ (45), a comparison of that $E[\tilde{V}]$ with the one defined by (41) in Example 1 reveals that since $\hat{D}$ (50) is less than $(a/3)\frac{\ln(v_{\max}/v_{\min})}{v_{\max}-v_{\min}}$ (i.e., $\hat{D}$ in Example 1) for all $v_{max} > v_{\min} > 0$, the $E[\tilde{V}]$ obtained for the uniformly distributed V is always smaller than its counterpart for the V that is defined by (48). This is consistent with the intuitive expectations because when $V = v_{\min} + ((v_{max} - v_{\min})/a)\,|x_s - x_d|$, the possibility of moving long distances with low speeds becomes zero. On the other hand, for the original random waypoint mobility model, since V is not directly proportional to D, lower speeds might be selected for longer distances and as a result, expected value of the long-run speed decreases. It should be also noted that, as $E[T_p] \to 0$ and $v_{\max} \to v_{\min}$, $E[\tilde{V}]$ converges to v_{min} for both choices of V, which is also expected because it corresponds to the scenario where mobile travels with fixed speed v_{min} at all times without pausing at any destination.

The other extreme case of interest for this example is $\sigma \to \infty$, which simplifies to the scenario where V is uniformly distributed in $[v_{\min}, v_{max}]$. Therefore, we conclude that, if $f_{V|X_s,X_d}$ is defined according to (42), the lower bound for $E[\tilde{V}]$ is given by (41) in Example 1, and the upper bound for it is given by (45) with the $\hat{D}$ defined as in (50). Obviously, the difference between these bounds decreases as $E[T_p] \to \infty$, or $v_{\min} \to v_{\max}$.

B. Variants of mobility characterizations done by $f_{X_d|X_s}$

Example 3: As a basic example of a scheme where distribution of X_d is dependent on X_s, consider a scenario where the closed region $R = [0, a]$ is partitioned into two subregions $R_1 = [0, c)$ and $R_2 = [c, a)$ such that $0 < c < a$. In this setting, when the starting point $X_s \in R_1$, the destination point X_d for that epoch will be distributed uniformly over either R_2 or R_1 with respective probabilities α and $(1-\alpha)$ where $0 < \alpha < 1$. Similarly, if $X_s \in R_2$, then X_d will be distributed uniformly over either R_1 or R_2 with probabilities β and $(1-\beta)$, respectively where $0 < \beta < 1$. Hence, the stochastic density kernel $f_{X_d|X_s}$ will be formulated by

$$f_{X_d|X_s}(x_d|x_s)=\begin{cases} \frac{1-\alpha}{c}, & \text{if } x_s \in [0,c) \text{ and } x_d \in [0,c) \\ \frac{\alpha}{a-c}, & \text{if } x_s \in [0,c) \text{ and } x_d \in [c,a) \\ \frac{\beta}{c}, & \text{if } x_s \in [c,a) \text{ and } x_d \in [0,c) \\ \frac{1-\beta}{a-c}, & \text{if } x_s \in [c,a) \text{ and } x_d \in [c,a) \\ 0, & \text{otherwise} \end{cases} \tag{51}$$

where $0 < \alpha < 1$ and $0 < \beta < 1$.

Now, let X_{d_k} denote the X_d (i.e. destination point) of the kth movement epoch. Then, based on the definition of $f_{X_d|X_s}$ given above (51), we can construct the DTMC $\{X_{d_k}, k \in \mathbf{N}\}$ with states that represent the subregions R_1, R_2, and a transition probability matrix A given by

$$A = \begin{array}{c} R_1 \\ R_2 \end{array} \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \tag{52}$$

Obviously, the DTMC $\{X_{d_k}, k \in \mathbf{N}\}$ governs the decisions of X_d at consecutive movement epochs, and in order to solve $f_{X_s}(x_d)$ uniquely from integral equation (9), it must satisfy the conditions of ergodicity at fist. Therefore, α or β can not be equal to 0 or 1, which is already required in the formulation of $f_{X_d|X_s}$.

Hence, by applying the integral equation defined in (9), we can derive the $f_{X_s}(x_d)$ for this example as follows:

$$f_{X_s}(x_d) = \begin{cases} \frac{1-\alpha}{c}\int_0^c f_{X_s}(x_s)dx_s \\ +\frac{\beta}{c}\int_c^a f_{X_s}(x_s)dx_s, & \text{if } x_d \in [0,c) \\ \frac{\alpha}{a-c}\int_0^c f_{X_s}(x_s)dx_s \\ +\frac{1-\beta}{a-c}\int_c^a f_{X_s}(x_s)dx_s, & \text{if } x_d \in [c,a) \end{cases} \tag{53}$$

which implies

$$f_{X_s}(x_d) = \begin{cases} k_1, & \text{if } x_d \in [0,c) \\ k_2, & \text{if } x_d \in [c,a) \end{cases} \tag{54}$$

for some constants k_1 and $k_2 \in \mathbf{R}$.

Now, let π_A denote the steady state distribution of the DTMC $\{X_{d_k}, k \in \mathbf{N}\}$ with transition probability A. Observe that $\pi_A = [\frac{\beta}{(\alpha+\beta)}, \frac{\alpha}{(\alpha+\beta)}]$, and since $\{X_{d_k}, k \in \mathbf{N}\}$ determines the subregion X_d is located, we get

$$\int_0^c f_{X_s}(x_d)dx_d = \frac{\beta}{(\alpha+\beta)}, \int_c^a f_{X_s}(x_d)dx_d = \frac{\alpha}{(\alpha+\beta)} \tag{55}$$

which concludes

$$f_{X_s}(x_d) = \begin{cases} \frac{\beta}{(\alpha+\beta)}\frac{1}{c}, & \text{if } x_d \in [0,c) \\ \frac{\alpha}{(\alpha+\beta)}\frac{1}{(a-c)}, & \text{if } x_d \in [c,a) \end{cases} \tag{56}$$

Next, we focus on a more generic form for this scenario. Consider a partitioning of the region $R = [0, a]$ into M subregions $R_i = [a_i, a_{i+1})$, $i = 1, \ldots, M$ such that $a_{i+1} > a_i$ with $a_1 = 0,\ a_{M+1} = a$, and let the stochastic density kernel be defined by

$$f_{X_d|X_s}(x_d|x_s)=\begin{cases} \frac{A_{i,j}}{a_{j+1}-a_j}, & \text{if } x_s \in R_i, \text{ and } x_d \in R_j, \\ & i,j = 1,\ldots,M \\ 0, & \text{otherwise} \end{cases} \tag{57}$$

where $A_{i,j}$ denote the probability of selecting X_d *uniformly* in subregion R_j given that X_s is located in subregion R_i.

Similar to the discussions for the solution of the integral equation given by (53), since the function $f_{X_d|X_s}(x_d|x_s)$ is independent from x_d in all of the different subregions for x_s, $f_{X_s}(x_d)$ will be equal to a constant value in all of the subregions R_i, $i = 1, \ldots, M$, as in (54). Therefore, if the DTMC $\{X_{d_k}, k \in \mathbf{N}\}$ with the $M \times M$ transition probability matrix $A = [A_{i,j}]$ is irreducible and aperiodic, then the stationary pdf of the destination points is given by

$$f_{X_s}(x_d) = \begin{cases} \frac{\pi_{A_i}}{a_{i+1}-a_i}, & \text{if } x_d \in R_i,\ i = 1,\ldots,M \\ 0, & \text{otherwise} \end{cases} \tag{58}$$

where $\pi_A = [\pi_{A_1}, \ldots, \pi_{A_M}]$ is the solution of the linear system $\pi_A A = \pi_A$, $\|\pi_A\|_1 = 1$.

As an application of this scenario, we focused on the one-dimensional version of the random direction model described in [31]. In this model, nodes are restricted to move between the destinations that are located at the ε neighborhood of boundaries. After reaching the destination, mobile pauses for a specified amount of time, and travels to a new destination,

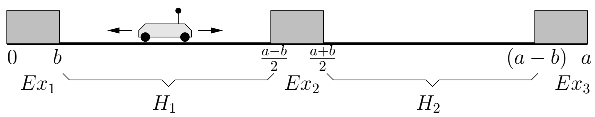

Fig. 3. Highway scenario for Example 4.

which is also located at the ε neighborhood of boundaries. Similar to the random waypoint mobility model, for each movement epoch, V is selected independently from $|X_s - X_d|$.

Now, in order to capture this model with the $f_{X_d|X_s}$ defined by (57) on a one-dimensional topology, we have to set $M = 3$, and divide R into subregions $R_1 = [0, \varepsilon)$, $R_2 = [\varepsilon, a - \varepsilon)$, and $R_3 = [a - \varepsilon, a)$. Since, the stochastic matrix A must be irreducible and aperiodic, we define it by

$$A = \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 0 & \epsilon & 1-\epsilon \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1-\epsilon & \epsilon & 0 \end{bmatrix} \tag{59}$$

where $0 < \epsilon < 1$. Obviously, since ϵ cannot be equal to 0, mobile terminals may select destination points located at R_2. However, as $\epsilon \to 0$, the possibility of this case diminishes, and we reach to desired scenario.

Hence, after obtaining the $f_{X_s}(x_d)$ from (58) for a nonzero ϵ, applying Theorem 1, and finally, by taking the limit of the result as $\epsilon \to 0$, we derived the following for the long-run location distribution of this mobility model:

$$f_X(x) = \begin{cases} \frac{E[T_p]/(2\varepsilon) + E[1/V]x/\varepsilon}{E[T_p] + \hat{D}}, & \text{if } x \in [0, \varepsilon) \\ \frac{E[1/V]}{E[T_p] + \hat{D}}, & \text{if } x \in [\varepsilon, a - \varepsilon) \\ \frac{E[T_p]/(2\varepsilon) + E[1/V](a-x)/\varepsilon}{E[T_p] + \hat{D}}, & \text{if } x \in [a - \varepsilon, a) \end{cases} \tag{60}$$

where $\hat{D} = E[1/V](a - \varepsilon)$, and $E[T_p]$ is the expected pause time spent at the destinations. Notice that, $f_X(x)$ converges to $\frac{1}{a}$ as $E[T_p] \to 0$ and $\varepsilon \to 0$.

Before proceeding to a more sophisticated scenario, we would like to emphasize an important issue about the usage of the stochastic density kernel $f_{X_d|X_s}(x_d|x_s)$ for mobility characterization. Now observe that $f_{X_d|X_s}$ provides a mechanism to accumulate the consecutive choices of destinations to subregions inside R. The transitions between the subregions can be also controlled by the transition probability matrix A we defined above. However, $f_{X_d|X_s}$ can not be employed in controlling the direction of the mobile terminal at consecutive movement epochs. For example, on the region $R = [0, a]$, our formulation can not be used to capture a case where mobile selects the destinations towards the point a, with higher probability for each movement epoch. In order to have a probabilistic mechanism to control the direction, we must extend the mobility model with an underlying modulating Markov chain that controls direction by making transitions at the embedded times at which a new movement epoch starts. This is doable for the discretisized version of the mobility formulation. However, it will never end up with tractable closed form expressions like the ones we presented by Lemmas 2 and 3.

Example 4: Consider the partitioning of the region $R = [0, a]$ shown in Fig. 3 where mobile terminals are expected to move between destinations located in the subregions Ex_i, $i = 1, 2, 3$, without pausing at the subregions H_1 and H_2.

From a practical point of view, this partitioning can be considered as a highway scenario where Ex_i and H_i represent exit areas and highway segments, respectively. The exit areas can be also considered as hotspots where mobile accumulate with higher probability. Hence, for the purpose of using our $< f_{X_d|X_s}, f_{V|X_s,X_d}, f_{T_p|X_d} >$ mobility characterization approach to capture a highway scenario that is composed of movement epochs between exit ares or hotspots, suppose that if $X_s \in Ex_i$, then X_d will be uniformly distributed either over Ex_j, for $j \neq i$, or over H_j, $j = 1, 2$, with respective probabilities α and $1/2 - \alpha$ where $0 < \alpha < 1/2$. Notice that, as $\alpha \to 1/2$, the possibility of a movement epoch to start from a highway segment, or to pause at somewhere on a highway segment becomes negligible. Furthermore, assume that if $X_s \in H_i$, then X_d will be uniformly distributed over either Ex_i or Ex_{i+1} with equal probabilities. Thus, the stochastic density kernel $f_{X_d|X_s}$ will be given by

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} \frac{\alpha}{b}, & \text{if } x_s \in Ex_i \text{ and } x_d \in Ex_j, \\ & i \neq j \\ \frac{(1/2)-\alpha}{(a-3b)/2}, & \text{if } x_s \in Ex_i \text{ and } x_d \in H_j \\ \frac{1/2}{b}, & \text{if } x_s \in H_i \text{ and } x_d \in Ex_j, \\ & j = i, i+1 \\ 0, & \text{otherwise} \end{cases} \tag{61}$$

where $0 < \alpha < 1/2$

Based on this definition of $f_{X_d|X_s}$, the transition probability matrix A corresponding the DTMC $\{X_{d_k}, k \in \mathbf{N}\}$ is

$$A = \begin{array}{c} Ex_1 \\ H_1 \\ Ex_2 \\ H_2 \\ Ex_3 \end{array} \begin{bmatrix} 0 & \frac{1}{2}-\alpha & \alpha & \frac{1}{2}-\alpha & \alpha \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \alpha & \frac{1}{2}-\alpha & 0 & \frac{1}{2}-\alpha & \alpha \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \alpha & \frac{1}{2}-\alpha & \alpha & \frac{1}{2}-\alpha & 0 \end{bmatrix} \tag{62}$$

Observe that, A satisfies the conditions of ergodicity if and only if $\alpha \neq 1/2$, which is also required by the definition of $f_{X_d|X_s}$.

Hence, by applying the result given by equation (58) for the $f_{X_d|X_s}$ of the form (57), we get

$$f_{X_s}(x_d) = \begin{cases} \frac{1+2\alpha}{8(1-\alpha^2)b}, & \text{if } x_d \in Ex_i, \ i = 1, 3 \\ \frac{1}{4(1-\alpha^2)b}, & \text{if } x_d \in Ex_2 \\ \frac{1-2\alpha}{2(1-\alpha)(a-3b)}, & \text{if } x_d \in H_j, \ j = 1, 2 \end{cases} \tag{63}$$

Furthermore, since we want the terminals to pause at only exit areas, we decided on the following function for the expected pause times at the destinations

$$E[T_p|X_s] = \begin{cases} C, & \text{if } X_s \in Ex_i, \ i = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases} \tag{64}$$

where C is a constant > 0. In addition, we assume that

$$V = v_{\min} + \frac{(v_{\max} - v_{\min})}{a} D \tag{65}$$

where $D = |X_s - X_d|$.

Based to the mobility characterization parameters we described, we generated k_X and $\hat{D}$ defined in Theorem 1 by the dividing the ranges of the double integration operations

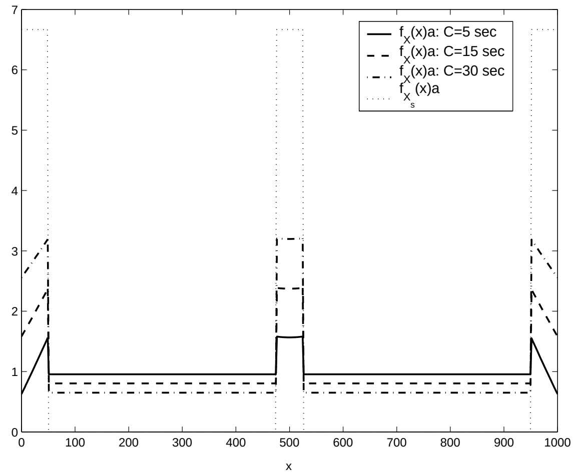

Fig. 4. Comparison of f_X and f_{X_s} as $\alpha \to 1/2$ for Example 4. ($a = 1000\,\mathrm{m}$, $b = 50\,\mathrm{m}$, $v_{\min} = 1$ m/s, $v_{max} = 20$ m/s, $C \in \{5, 10, 15\}$sec)

confidently to the subregions defined above. After this, we derived the limiting expressions of them as $\alpha \to 1/2$, and finally we obtained the pdf f_X for that limiting case (i.e., the case where destinations are only selected at the exit areas). Since V is dependent on D, the final form of f_X is not simple enough to fully present here. However, plots of f_X for different cases, and a graphical comparison of it with f_{X_s} can be found in Fig. 4. From Fig. 4, first observe that f_X and f_{X_s} are substantially different. This is expected because, during moving mobile terminal passes through highway segments and although they don't pause at highways, the proportion of time spent at highways locations increases as they move between exit areas. Furthermore, as the expected value of pause times at the exit areas (i.e. C) increases, the value of f_X at the highway segments decreases because they spend more time on the exit areas at the long-run. In addition, this example also shows that a performance analysis study that makes assumptions about the location distribution can not ignore the times spent on the highways that connect hotspots, or the subregions where mobile terminals accumulate with higher probability.

C. Modeling Acceleration

The obvious unrealistic characteristic of the movement behavior generated by our generalized approach of mobility modeling is that at the beginning of a movement epoch that had started at X_s and destined to X_d, the instantaneous speed, that is, speed at any instant of time, of a mobile terminal jumps from 0 to V abruptly implying an acceleration that is ∞ in magnitude. In addition, when mobile reaches to the destination, it decreases from V to 0 with a deceleration that is also ∞ in magnitude. However, in realistic situations, the magnitudes of acceleration and deceleration are finite, and a mobile terminal cannot immediately increase its instantaneous speed from 0 to V at the point X_s, and also immediately drop it from V to 0 at the point X_d. Clearly other random walk or random waypoint like mobility models that we have mentioned in Section I also possesses this unrealistic characteristic.

Now in order to remove this unrealistic movement behavior from our mobility formulation, assume that for each movement

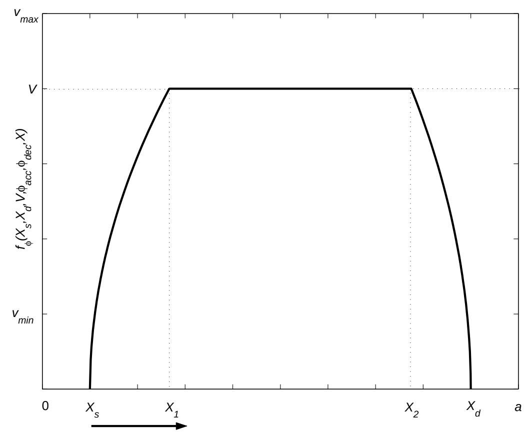

Fig. 5. Mobile reaches target speed V.

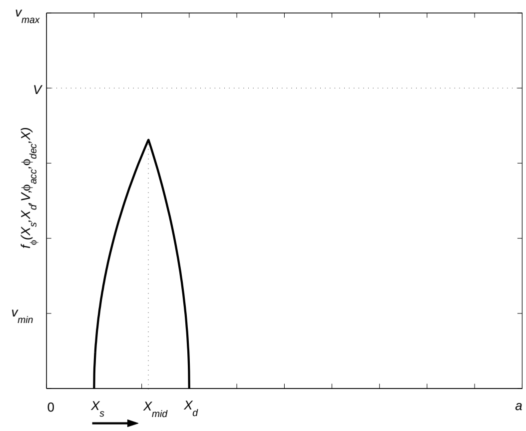

Fig. 6. Mobile slows down before reaching target speed V.

epoch, a mobile terminal increases its speed from 0 to V *uniformly* with an acceleration that has *constant* magnitude, travels at speed V for a distance, and when it gets close to destination, it decreases its speed from V to 0 uniformly with an deceleration that is also *constant* in magnitude. Let ϕ_{acc} and ϕ_{dec} denote the magnitudes of acceleration and deceleration, respectively. Before we proceed further in the analysis, we assume that the distance between the point X_d and the location at which mobile starts slowing down must be exactly equal to the distance required to decrease speed from V to 0 with a deceleration that is equal to ϕ_{dec} in magnitude (i.e., a symmetric environment). In addition, in the rest of this subsection, since terminals accelerate to and from speed V, which is drawn randomly from $[v_{\min}, v_{\max}]$, the random variable V will be also called as "target speed". Hence, let $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X)$ denote the speed of the mobile terminal at the point X for the movement epoch between X_s and X_d with target speed V, constant acceleration ϕ_{acc}, and constant deceleration ϕ_{dec}. Notice that acceleration becomes 0 when the terminal reaches target speed V. However, for some exceptional cases, the absolute distance between X_s and X_d can be so small that the mobile might be forced to decelerate before reaching to target speed V. In order to illustrate these characteristics, in Fig. 5 and Fig. 6, we focused on a single movement epoch between the points X_s and X_d where $X_d >$

X_s, and plotted the instantaneous speed of a terminal versus its location (i.e. X). Observe that when destination X_d is too close to X_s, mobile terminal cannot reach instantaneous speed V, which is selected as the speed of the movement interval between X_s and X_d, and has to decelerate after an acceleration period. More formally, for the case where $X_d > X_s$, let $X_1 = X_s + \frac{V^2}{2\phi_{acc}}$, and $X_2 = X_d - \frac{V^2}{2\phi_{dec}}$. Hence, if $X_1 \le X_2$, then

$$f_\phi(X_s,X_d,V,\phi_{acc},\phi_{dec},X)=\begin{cases}\sqrt{2\phi_{acc}\,(X-X_s)}, & X\in(X_s,X_1]\\ V, & X\in(X_1,X_2]\\ \sqrt{V^2-2\phi_{dec}\,(X-X_2)}, & X\in(X_2,X_d]\end{cases} \quad (66)$$

On the other hand, if $X_1 > X_2$ (i.e., mobile must slow down before reaching speed V), then

$$f_\phi(X_s,X_d,V,\phi_{acc},\phi_{dec},X)=\begin{cases}\sqrt{2\phi_{acc}\,(X-X_s)}, & X\in(X_s,X_{mid}]\\ \sqrt{2\phi_{dec}\,(X_d-X)}, & X\in(X_{mid},X_d]\end{cases} \quad (67)$$

where $X_{mid} = X_s + \frac{\phi_{dec}(X_d-X_s)}{\phi_{dec}+\phi_{acc}}$.

For the other case where $X_d < X_s$, let $X_1 = X_s - \frac{V^2}{2\phi_{acc}}$, and $X_2 = X_d + \frac{V^2}{2\phi_{dec}}$. Hence, if $X_1 \ge X_2$, then

$$f_\phi(X_s,X_d,V,\phi_{acc},\phi_{dec},X)=\begin{cases}\sqrt{2\phi_{acc}\,(X_s-X)}, & X\in(X_1,X_s]\\ V, & X\in(X_2,X_1]\\ \sqrt{V^2+2\phi_{dec}\,(X-X_2)}, & X\in(X_d,X_2]\end{cases} \quad (68)$$

However, if $X_1 < X_2$ (i.e., exceptional case), then

$$f_\phi(X_s,X_d,V,\phi_{acc},\phi_{dec},X)=\begin{cases}\sqrt{2\phi_{acc}\,(X_s-X)}, & X\in(X_{mid},X_s]\\ \sqrt{2\phi_{dec}\,(X-X_d)}, & X\in(X_d,X_{mid}]\end{cases} \quad (69)$$

where $X_{mid} = X_s - \frac{\phi_{dec}(X_s-X_d)}{\phi_{dec}+\phi_{acc}}$.

We note that if $\phi_{acc} = \infty$ and $\phi_{dec} = \infty$, then $X_1 = X_s$ and $X_2 = X_d$ for all of the cases we defined above, and consequently $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = V$ at all points between X_s and X_d.

It is now apparent from these formulations that in order to capture acceleration-deceleration characteristics of vehicles, mobility formulation must keep the information about the starting point (i.e., X_s) of each movement epoch that is destined to the point X_d. Since we employ the stochastic density kernel $f_{X_d|X_s}$ in mobility characterization, this requirement has already been satisfied.

Now in order to formulate the long-run location and speed distributions according to the acceleration and deceleration parameters, we first need to extend the results given for the discretisized mobility formulation. Returning back to Lemma 2, observe from the formulation of $\boldsymbol{\pi}_{i,j}$ (13) (i.e., the probability of being in cell c_i and moving towards cell c_j at the steady-state) that the steady-state probability of being at c_i for a movement epoch that had started at c_ℓ and destined to c_j with a target speed of $z_r = r\Delta v$ is simply $\varphi_\ell\, \tau_{j|\ell}\, \nu_{r|\ell,j}/N$. Hence, the acceleration and deceleration characteristics can be easily incorporated into the formulation of p_i (18) given in Lemma 3 by substituting the z_r appearing inside the formulation of k_i (19) with the discretisized version of speed that can be achieved at cell c_i for a movement epoch that had started at c_ℓ and destined to c_j. Thus, using the limiting approach that was applied to derive the result presented in Theorem 1, it can be easily proven that, in order to capture acceleration-deceleration characteristics with the long-run location distribution, it is enough to replace $E[\frac{1}{V}|X_s = x_s, X_d = x_d]$ (24), which has the same value for all X between the points X_s and X_d, with

$$E[\frac{1}{V}|X_s = x_s, X_d = x_d, X = x] = \int_{v_{min}}^{v_{max}} \frac{dv}{f_\phi(x_s, x_d, v, \phi_{acc}, \phi_{dec}, x)}\, f_{V|X_s,X_d}(v|x_s, x_d), \quad (70)$$

and k_X (22) must be redefined by

$$k_X(x)=\int_0^x dx_d\int_x^a dx_s\, g_X(x_s, x_d, x) + \int_x^a dx_d\int_0^x dx_s\, g_X(x_s, x_d, x) \quad (71)$$

where

$$g_X(x_s, x_d, x)=f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) E[\tfrac{1}{V}|X_s=x_s,X_d=x_d,X=x] \quad (72)$$

Next, notice that when acceleration-deceleration formulation comes into the picture, since mobile accelerates (decelerates) to (from) target speed V, $\tilde{V}(t)$, that is, the speed of the mobile at time t, must be defined on the set $\{0\} \cup \{v|0 < v \le v_{max}\}$. Therefore, the distribution of $\tilde{V}$ (i.e., the random variable having the long-run distribution of $\tilde{V}(t)$) can only be determined by considering all possible target speeds $V \in [v_{\min}, v_{\max}]$ for a given movement epoch between X_s and X_d, and checking whether it is possible to have speed $\tilde{V}$ at a point X on the path between X_s and X_d. As a result, using the formulation of $\boldsymbol{\pi}_{i,j}$ (13), we obtained the following pdf for $\tilde{V}$, which was first defined in Theorem 2 for the infinite acceleration-deceleration case,

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p|0\le X_s\le a]\delta(\tilde{v})}{E[T_p|0\le X_s\le a]+\hat{D}}, & \text{if } \tilde{v} = 0 \\ \frac{\int_0^a dx\, k_{\tilde{V}}(x,\tilde{v})}{E[T_p|0\le X_s\le a]+\hat{D}}, & \text{if } \tilde{v} \in (0, v_{\max}] \end{cases}, \quad (73)$$

where $k_{\tilde{V}}(x, \tilde{v})$ is defined by (35), but the integrand of it, (i.e., $g_{\tilde{V}}(x_s, x_d, \tilde{v})$ (35)), is reformulated by

$$g_{\tilde{V}}(x_s, x_d, \tilde{v}, x) = f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \tilde{g}_\phi(x_s, x_d, \tilde{v}, \phi_{acc}, \phi_{dec}, x) \quad (74)$$

where

$$\tilde{g}_\phi(x_s, x_d, \tilde{v}, \phi_{acc}, \phi_{dec}, x) = \int_{v_{min}}^{v_{max}} dv f_{V|X_s,X_d}(v|x_s, x_d) \frac{1}{\tilde{v}} \mathbf{1}_{\{\tilde{v}=f_\phi(x_s,x_d,v,\phi_{acc},\phi_{dec},x)\}}, \quad (75)$$

which implies

$$E[\tilde{V}] = \frac{\bar{D}}{E[T_p|0 \le X_s \le a] + \hat{D}} \quad (76)$$

where $\bar{D}$ is simply given by (28), and $\hat{D}$ is again defined by (25), but its integrand is the k_X formulated above by (71).

At this point, it should be noted that since the function $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X)$ is determined according to the comparison of the variables X_1 and X_2, which are defined in terms of X_s, X_d, V, ϕ_{acc}, and ϕ_{dec}, it is very complicated to

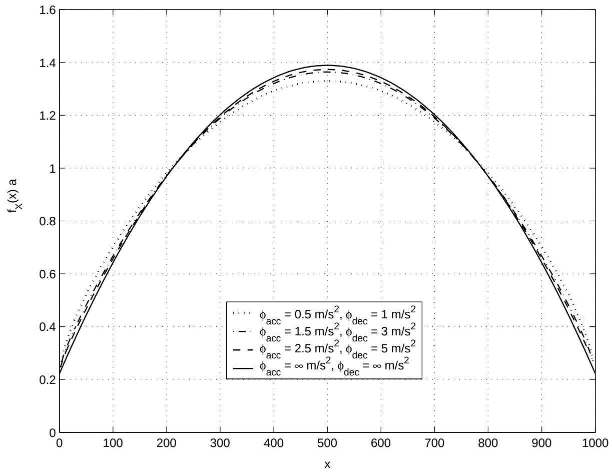

Fig. 7. f_X for Example 5. ($a = 1000\,$m, $v_{min} = 1\,$m/s, $v_{max} = 20\,$m/s, $E[T_p] = 15\,$sec)

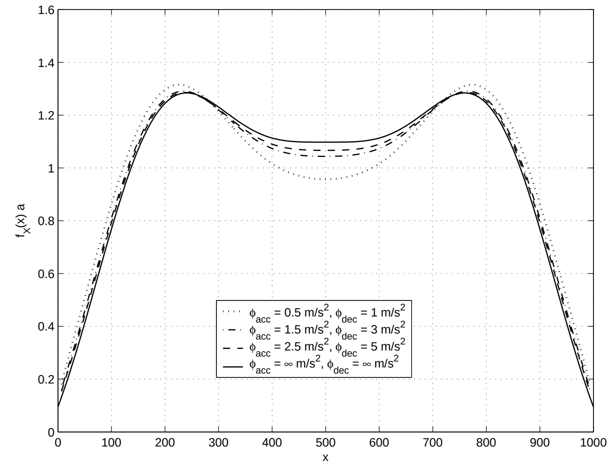

Fig. 8. f_X for Example 6. ($a = 1000\,$m, V is uniform in $[v_{min}, v_{max}]$, $v_{\min} = 1\,$m/s, $v_{max} = 20\,$m/s, $C = 10\,$sec)

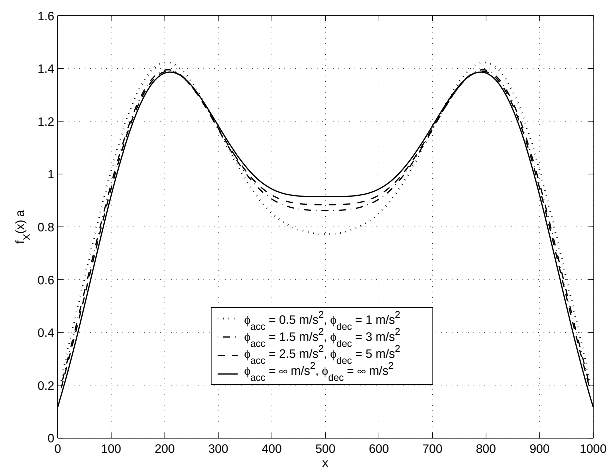

Fig. 9. f_X for Example 6. ($a = 1000\,$m, $f_{V|X_s,X_d}$ is given by (42), $\sigma = 5$, $v_{min} = 1\,$m/s, $v_{max} = 20\,$m/s, $C = 10\,$sec)

find a closed form expressions for k_X (71) even for the simplest nontrivial case (i.e., random waypoint mobility model). Therefore, in the following example scenarios, which are presented to demonstrate the effects of different acceleration-deceleration parameters on the long-run location distribution and expected value of speed at the long-run, we evaluated k_X, and also $\hat{D}$ using numerical integration methods.

Example 5: In this example, we focus on the original random waypoint mobility model (i.e., uniformly selected destination and speed, location independent pause time distribution). Fig. 7 depicts several f_X that are obtained for different acceleration-deceleration parameters.

First, observe that as ϕ_{acc} and ϕ_{dec} increases, the plot of f_X gets close to the plot of the case where $\phi_{acc} = \infty$ and $\phi_{dec} = \infty$, which is consistent with the intuitive expectations. Second, for reasonable values of acceleration and deceleration, the probability of the mobile terminal to be located at the center of the region is lower than the case of infinite acceleration and deceleration.

Example 6: For this example, we assume that the distribution of X_d is independent from X_s and is given by the following sinusoidal function

$$f_{X_d}(x_d) = \frac{3\pi(1 + \sin(3\pi x_d/a))}{a(2 + 3\pi)} \tag{77}$$

which has maximums at the points $a/6$ and $5a/6$, and minimum at $a/2$. Furthermore, we assume that

$$E[T_p|X_s = x_s] = a\,C\,f_{X_d}(x_s) \tag{78}$$

where $C > 0$, which implies $E[T_p|0 \leq X_s \leq a] \approx 1.31C$.

Observe that, these mobility characterization parameters can be used model a scenario where mobiles select the destinations around the points $a/6$ and $5a/6$ with higher probability, and pause for a longer amount of time around those locations.

In Fig. 8, we assumed V to be uniformly distributed in $[v_{\min}, v_{\max}]$, and plotted f_X for different acceleration and deceleration parameters. Clearly the first observation that we have made in Example 5 for the effects of the acceleration-deceleration on the random waypoint model is also valid for this scenario. However, it is clear that for this scenario, the difference between finite and infinite acceleration-deceleration cases becomes noticeable as acceleration-deceleration parameters decreases, especially at the center of the region.

For comparison purposes, we also concentrated on the case where $f_{V|X_s,X_d}$ is defined by the truncated exponential distribution defined in Example 2 by (42). Remember that, since $\mu(x_s, x_d)$ is linearly dependent to $|x_d - x_s|$, the possibility of selecting V directly proportional to $|X_d - X_s|$ increases as σ decreases. In Fig. 9, we set $\sigma = 5$ and plotted f_X for different acceleration-deceleration parameters. Notice that when V is proportional to the distance that is going to be traveled (i.e., $|X_s - X_d|$), the long-run location distribution becomes less sensitive to the acceleration-deceleration characteristics of vehicles. In addition, long-run proportion of times spent at the locations connecting hotspots that are accumulated around the points $a/6$ and $5a/6$ decreases when V is proportional to $|X_s - X_d|$. These are expected because in this scenario, mobility model do not assign a speed V for a movement epoch that is impossible to achieve, for example, high speed for a short distance, or a speed that is unrealistically low for long distance. We also note that, the experiments that are presented by Fig. 9, can be also done for less values of σ. However,

TABLE II
$E[\tilde{V}]$ FOR EXAMPLE 7

ϕ_{acc} (m/s^2)	ϕ_{dec} (m/s^2)	$E[T_p]$ (sec)	$E[\tilde{V}]$ (m/s) (v_{min}=1 m/s, v_{max}=20 m/s) $\sigma \to \infty$	$\sigma = 10$	$\sigma = 5$	$\sigma = 1$
0.5	1	0	5.03	5.31	5.71	6.39
1.5	3	0	5.82	6.15	6.62	7.41
2.5	5	0	6.02	6.37	6.84	7.65
∞	∞	0	6.34	6.70	7.17	8.00
0.5	1	5	4.68	4.92	5.26	5.84
1.5	3	5	5.35	5.63	6.02	6.67
2.5	5	5	5.52	5.81	6.20	6.86
∞	∞	5	5.79	6.09	6.48	7.15
0.5	1	15	4.10	4.26	4.55	4.97
1.5	3	15	4.61	4.82	5.09	5.56
2.5	5	15	4.74	4.95	5.23	5.69
∞	∞	15	4.93	5.15	5.42	5.88

since we are evaluating the k_X and $\hat{D}$ numerically, the cost of the numerical integration procedures increases as the p.d.f. $f_{V|X_s,X_d}$ converges to the form given by (47) (i.e., the unit-impulse function at the point $\mu(x_s, x_d)$).

Example 7: As a final example, we concentrated on the measure $E[\tilde{V}]$, that is, expected speed at the long-run, which is formulated by (76) for the finite acceleration-deceleration parameters. In order to also analyze the case that captures the method of determining V according to the distance that is going to be traveled, we considered the mobility parameters of Example 2.

Recall that, the $f_{V|X_s,X_d}$ defined by (42) in Example 2 converges to the uniform distribution in $[v_{\min}, v_{\max}]$ as $\sigma \to \infty$. Hence, we evaluated $E[\tilde{V}]$ for four different values of σ, and for infinite and various finite acceleration-deceleration parameters. Results are shown in Table II.

As it can be seen in Table II, the values of $E[\tilde{V}]$ for the finite acceleration-deceleration parameters are always less than their counterparts that are evaluated by assuming acceleration and deceleration to be infinite. Obviously, the difference between them increases as the parameters ϕ_{acc} and ϕ_{dec} decreases. On the other hand, the gap between the $E[\tilde{V}]$ obtained for the same infinite and finite pairs of ϕ_{acc} and ϕ_{dec} decreases, as $E[T_p]$ increases, which is expected because the proportion of time $\tilde{V}$ possesses zero speed also increases. In addition, for given values of ϕ_{acc}, ϕ_{dec}, and $E[T_p]$, a comparison of the value of $E[\tilde{V}]$ with its counterpart for the infinite ϕ_{acc}, ϕ_{dec} case reveals out that the difference between them is more or less the same for all values of σ considered. From this observation, we conclude that if $D_\phi(X_s, X_d, V)$ denotes the total distance traveled while accelerating and decelerating during a movement epoch between X_s and X_d with a target speed of V, then the proportion $\frac{D_\phi(X_s,X_d,V)}{|X_s-X_d|}$ averaged over all possible X_s, X_d, and V is rather insensitive to the choice of σ. Hence, even if V is determined according to the distance that is going to be traveled with high probability, there will always be periods of acceleration and deceleration that affects the value of $E[\tilde{V}]$.

Consequently, the results presented in Table II shows that if a performance measure of interest evaluated for an wireless ad hoc network is dependent on the expected speed at the long run (i.e., $E[\tilde{V}]$), then the acceleration-deceleration characteristics of the mobile terminals must be captured by the mobility model.

IV. CONCLUSIONS

For ad hoc wireless networks, we proposed a generalized random mobility model capable of capturing several scenarios, including hotspots and displacement dependent speed distributions. The analytical framework we presented for the long-run analysis of this generic mobility model over one-dimensional mobility terrains provided closed form expressions for the long-run location and speed distributions. We also provided an extension on our results so that they can be used to examine the effects of acceleration characteristics of vehicles on the long-run location and speed distributions. Our example scenarios verify the usefulness of our analytical framework for the mobility analysis and yield significant insights into how realistic mobility scenarios can be brought into the capacity analysis of wireless ad hoc networks. Future work will consider the extension of these results to two-dimensional regions.

APPENDIX

Proof of Lemma 1

Proof: Since the integral equation (9) is uniquely solvable, a movement epoch starts from the cell c_i with a nonzero probability φ_i at the steady-state. Hence, we can concentrate on the reachable states from the states of the form $(c_i, 0)$.

Now from the ordering of the states given in partition $\mathcal{S}_i$ (5), and the transition probabilities specified in Table I, it can be observed that it is possible to jump from a pause state $(c_i, 0)$ to all of the moving states of the form $(c_i, c_j, z_r, 1)$ with some nonzero probability. Consequently, if the states of the form $(c_i, 0)$ is reachable from other pausing states $(c_j, 0)$, where $i \neq j$, then the Markov chain becomes irreducible. This is easy to prove because according to the transition probabilities of the Markov chain, the process jumps to the state $(c_i, c_j, z_r, 1)$ from state $(c_i, 0)$ with probability $\frac{\tau_{j|i}}{1-\tau_{i|i}}\nu_{r|i,j}$ and enters the state $(c_j, 0)$ with probability 1 in $|i-j|$ transitions.

Hence, since the Markov chain is irreducible, all states are periodic with the same period, or else all states are aperiodic. Without loss of generality, assume we want to find out whether the state $(c_0, c_1, z_1, 1)$ is aperiodic or not. Due to the rules of transitions given in Table I, if the mobile selects c_0 as destination after reaching c_1, then the process may go back to $(c_0, c_1, z_1, 1)$ in $d_1 = 4$ transitions. However, after pausing at c_1 since n is assumed to be greater than 1, it may also choose c_2 as destination cell with some nonzero probability. Suppose it selects c_2 as destination. If it selects c_0 as target again when it reaches c_2, then process may return back to $(c_0, c_1, z_1, 1)$ in $d_2 = 7$ steps. On the other hand, if it chooses at first c_1 and then c_0 as destination when it is located at c_2, then it may go back to $(c_0, c_1, z_1, 1)$ after leaving it in $d_3 = 8$ steps. Since the greatest common divisor of d_1, d_2, and d_3 is 1, the state $(c_0, c_1, z_1, 1)$ becomes aperiodic. Therefore, the Markov chain is aperiodic and the proof completes. ■

Proof of Lemma 2

Proof: The proof is by direct substitution. First, observe that

$$\begin{aligned}
\boldsymbol{\pi}_0 A_1^{(0)} &= [0, \varphi_0\tau_{1|0}\boldsymbol{\nu}_{m|0,1}, \varphi_0\tau_{2|0}\boldsymbol{\nu}_{m|0,2}, \dots, \\
&\quad \varphi_0\tau_{n-1|0}\boldsymbol{\nu}_{m|0,n-1}], \\
\boldsymbol{\pi}_1 A_2^{(1)} &= [\varphi_0(1-\tau_{0|0}), 0, \dots, 0], \\
\boldsymbol{\pi}_0 A_1^{(0)} &+ \boldsymbol{\pi}_1 A_2^{(1)} = \boldsymbol{\pi}_0. \qquad (79)
\end{aligned}$$

In the same way,

$$\begin{aligned}
\boldsymbol{\pi}_{n-1} A_1^{(n-1)} &= [\varphi_{n-1}\tau_{0|n-1}\boldsymbol{\nu}_{m|n-1,0}, \varphi_{n-1}\tau_{1|n-1}\boldsymbol{\nu}_{m|n-1,1}, \\
&\quad \dots, \varphi_{n-1}\tau_{n-2|n-1}\boldsymbol{\nu}_{m|n-1,n-2}, 0], \\
\boldsymbol{\pi}_{n-2} A_0^{(n-2)} &= [0, \dots, 0, \varphi_{n-1}(1-\tau_{n-1|n-1})], \\
\boldsymbol{\pi}_{n-1} A_1^{(n-1)} &+ \pi_{n-2} A_0^{(n-2)} = \boldsymbol{\pi}_{n-1} \qquad (80)
\end{aligned}$$

For $1 \le i \le n-2$, observe the following

$$\begin{aligned}
\boldsymbol{\pi}_i A_1^{(i)} &= [\varphi_i\tau_{0|i}\boldsymbol{\nu}_{m|i,0}, ..., \varphi_i\tau_{i-1|i}\boldsymbol{\nu}_{m|i,i-1}, 0, \\
&\quad \varphi_i\tau_{i+1|i}\boldsymbol{\nu}_{m|i,i+1}, ..., \varphi_i\tau_{n-1|i}\boldsymbol{\nu}_{m|i,n-1}], \\
\boldsymbol{\pi}_{i-1} A_0^{(i-1)} &= [0, \dots, 0, \sum_{\ell=0}^{i-1} \varphi_\ell\tau_{i|\ell}, \sum_{\ell=0}^{i-1} \varphi_\ell\tau_{i+1|\ell}\boldsymbol{\nu}_{m|\ell,i+1}, \\
&\quad \dots, \sum_{\ell=0}^{i-1} \varphi_\ell\tau_{n-1|\ell}\boldsymbol{\nu}_{m|\ell,n-1}], \\
\boldsymbol{\pi}_{i+1} A_2^{(i+1)} &= [\sum_{\ell=i+1}^{n-1} \varphi_\ell\tau_{0|\ell}\boldsymbol{\nu}_{m|\ell,0}, \dots, \sum_{\ell=i+1}^{n-1} \varphi_\ell\tau_{i-1|\ell}\boldsymbol{\nu}_{m|\ell,i-1}, \\
&\quad \sum_{\ell=i+1}^{n-1} \varphi_\ell\tau_{i|\ell}, 0, \dots, 0], \\
\boldsymbol{\pi}_{i-1} A_0^{(i-1)} &+ \boldsymbol{\pi}_i A_1^{(i)} + \boldsymbol{\pi}_{i+1} A_2^{(i+1)} = \boldsymbol{\pi}_i. \qquad (81)
\end{aligned}$$

Finally, using equalities (79), (80), (81), and normalizing each $\boldsymbol{\pi}_i$, $i = 0, \dots, n-1$, with $N = \sum_{i=0}^{n-1} \|\boldsymbol{\pi}_i\|_1$, it is easy to see that $\boldsymbol{\pi} P = \boldsymbol{\pi}$, with $\|\boldsymbol{\pi}\|_1 = 1$ holds, which concludes the proof. ■

Proof of Lemma 3

Proof: According to the state partitioning given in (5), all of the states in partition $\mathcal{S}_i$ are located at cell c_i. Hence, p_i is simply

$$p_i = \sum_{s \in \mathcal{S}_i} P_s, \quad i = 0, \dots, n-1 \qquad (82)$$

Using the above equation, the $\boldsymbol{\pi}_{i,j}$ formulated in (13), and expected state holding times derived in (14), and (15), we obtained the p_i given as in (83), located at the bottom of the page.

Now, defining

$$\begin{aligned}
k_i &= \sum_{j=0}^{i-1} \sum_{\ell=i}^{n-1} \varphi_\ell\, \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j} \\
&+ \sum_{j=i+1}^{n-1} \sum_{\ell=0}^{i} \varphi_\ell\, \tau_{j|\ell} \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j}
\end{aligned}$$

the formula given in (83) simplifies to form given by (18). ■

Proof of Theorem 1

Proof: To derive this result, we first formulate the equations given in (18), (19), and (20) in terms of f_X, f_{X_s}, $f_{X_d|X_s}$, and $f_{V|X_s,X_d}$, and then take the limit of the expression as $n \to \infty$ (i.e., $\Delta x \to 0$), and $m \to \infty$ (i.e., $\Delta v \to 0$).

First, for small Δx and Δv observe the following:

$$\begin{aligned}
p_i &= f_X(x_i^*)\,\Delta x, & (84)\\
\varphi_i &= f_{X_s}(x_i^*)\,\Delta x, & (85)\\
\tau_{j|i} &= f_{X_d|X_s}(x_j^*|X_S \in c_i)\,\Delta x, & (86)\\
\nu_{r|\ell,j} &= f_{V|X_s,X_d}(v_r^*|X_s \in c_\ell, X_d \in c_j)\,\Delta v & (87)
\end{aligned}$$

where the numbers x_i^*, x_j^*, and v_r^* are chosen arbitrarily within the subintervals $[i\Delta x, (i+1)\Delta x)$, $[j\Delta x, (j+1)\Delta x)$, and $[r\Delta v, (r+1)\Delta v)$, respectively, $i, j = 0, \dots, n-1$, and $r = 1, \dots, m$. By inserting these approximations back to (18), (19), and (20), and canceling Δx from both sides of the equation, we obtained

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$$p_i = \frac{\varphi_i(1-\tau_{i|i})E[T_{P_i}] + \Big(\sum_{j=0}^{i-1}\sum_{\ell=i}^{n-1} \varphi_\ell\, \tau_{j|\ell}\, \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j} + \sum_{j=i+1}^{n-1}\sum_{\ell=0}^{i} \varphi_\ell\, \tau_{j|\ell}\, \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j}\Big)\Delta x}{\sum_{\imath=0}^{n-1} \varphi_\imath(1-\tau_{\imath|\imath})E[T_{P_\imath}] + \sum_{\imath=0}^{n-1}\Big(\sum_{j=0}^{\imath-1}\sum_{\ell=\imath}^{n-1} \varphi_\ell\, \tau_{j|\ell}\, \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j} + \sum_{j=\imath+1}^{n-1}\sum_{\ell=0}^{\imath} \varphi_\ell\, \tau_{j|\ell}\, \sum_{r=1}^{m} \frac{1}{z_r}\, \nu_{r|\ell,j}\Big)\Delta x} \qquad (83)$$

$$f(x_i^*) = \frac{f_{X_s}(x_i^*)\,(1 - f_{X_d|X_s}(x_i^*|X_S \in c_i)\,\Delta x)E[T_{p_i}] + k_i^*}{\sum_{\imath=0}^{n-1} E[T_{p_\imath}] f_{X_s}(x_\imath^*)\,\Delta x - \sum_{\imath=0}^{n-1} E[T_{p_\imath}] f_{X_d|X_s}^2(x_\imath^*|X_S \in c_\imath)\,(\Delta x)^2 + \bar{D}_n^*} \qquad (87)$$

$$\begin{aligned}
k_i^* &= \sum_{j=0}^{i-1}\sum_{\ell=i}^{n-1} f_{X_s}(x_\ell^*)\,\Delta x\; f_{X_d|X_s}(x_j^*|X_S \in c_\ell)\,\Delta x\; \sum_{r=1}^{m} \frac{1}{z_r}\, f_{V|X_s,X_d}(v_r^*|X_s \in c_\ell, X_d \in c_j)\,\Delta v \\
&+ \sum_{j=i+1}^{n-1}\sum_{\ell=0}^{i} f_{X_s}(x_\ell^*)\,\Delta x\; f_{X_d|X_s}(x_j^*|X_S \in c_\ell)\,\Delta x\; \sum_{r=1}^{m} \frac{1}{z_r}\, f_{V|X_s,X_d}(v_r^*|X_s \in c_\ell, X_d \in c_j)\,\Delta v \qquad (88)
\end{aligned}$$

the equation given in (87), shown at the bottom of the page where k_i^* is defined by (88), and

$$\hat{D}_n^* = \sum_{\imath=0}^{n-1} k_i^* \tag{90}$$

To complete the proof, we have to take the limit of (87) as $\Delta x \to 0$. Clearly, $(1 - f_{X_d|X_s}(x_i^*|X_S \in c_i)\,\Delta x) \to 1$ as $\Delta x \to 0$. Now, observe the following:

$$\lim_{\Delta x \to 0} E[T_{p_i}] =$$

$$\lim_{\Delta x\to 0} \int_0^\infty dt_p\, \frac{\Pr\{T_p > t_p,\ i\Delta x \le X_s < (i+1)\Delta x\}}{\Pr\{i\Delta x \le X_s < (i+1)\Delta x\}}$$

$$= \lim_{\Delta x\to 0} \int_0^\infty dt_p \frac{\int_{t_p}^\infty dt \int_{i\Delta x}^{(i+1)\Delta x} du\, f_{T_p,X_s}(u,t)}{\int_{i\Delta x}^{(i+1)\Delta x} du\, f_{X_s}(u)}$$

$$= \int_0^\infty dt_p \frac{\int_{t_p}^\infty dt\, f_{T_p,X_s}(x_i^*,t)}{f_{X_s}(x_i^*)}$$

$$= \int_0^\infty dt_p \int_{t_p}^\infty dt\, f_{T_p|X_s}(x_i^*,t)$$

$$= \int_0^\infty dt_p \Pr\{T_p > t_p | X_s = x_i^*\} = E[T_P|X_s = x_i^*], \tag{91}$$

$$\lim_{\Delta x\to 0} \sum_{\imath=0}^{n-1} E[T_{p_\imath}] f_{X_s}(x_i^*)\,\Delta x = \int_0^a E[T_p|X_s = u] f_{X_s}(u)\,du$$

$$= E[T_p|0 \le X_s \le a], \tag{92}$$

and

$$\lim_{\substack{\Delta x\to 0\\ \Delta v\to 0}} k_i^*$$

$$= \int_0^{x_i^*} dx_d \int_{x_i^*}^{a} dx_s\, f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) E[\tfrac{1}{V}|X_s{=}x_s, X_d{=}x_d]$$

$$+ \int_{x_i^*}^{a} dx_d \int_0^{x_i^*} dx_s\, f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) E[\tfrac{1}{V}|X_s{=}x_s, X_d{=}x_d] \tag{93}$$

In addition, the term $\sum_{\imath=0}^{n-1} E[T_{p_\imath}] f^2_{X_d|X_s}(x_i^*|X_S \in c_\imath)\,(\Delta x)^2$ converges to 0 as $\Delta x \to 0$. Combining limits (91), (92), (93), and substituting x_i^* with x, we obtained the result presented in Theorem 1. ∎

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