PreCalculus Writing Assignment

Solve the inequality below for all $x\ in\ [0,2\pi]$:

$$|\sin(x)| \geq |\cos(x)|$$

Explain and illustrate at least three solutions. For instance, in the "window" $[0,2\pi] \times [-1,1]$ carefully consider the graphs of both $\sin(x)$ and $\cos(x)$. Can you simplify the question with algebra or trigonometry? Your calculator or a computer algebra system will help you visualize solutions.

SAMPLE #1

Calculus I Writing Assignment

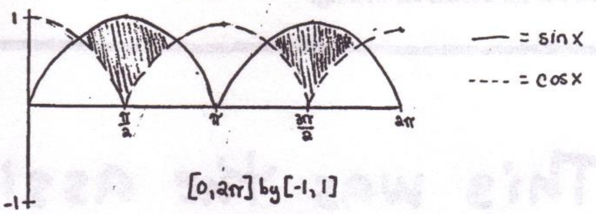
"A special three-in-one volume!"

[redacted] Section D

The problem: find where the absolute value of the sine of "x" is greater than or equal to the absolute value of the cosine of "x" where "x" is between 0 and 2 times pi.

Method One:

To solve the above inequality, I first decided to use a graph. This would help me better visualize the problem situation and hopefully lead to other solution possibilities. Thus, I graphed the equations y = abs(sin x), and y = abs(cos x) in my graphing calculator. Of course, x is an angle in a right triangle (besides the right angle), and abs stands for absolute value. The graph below was the result.



By reading this graph, we can easily see where the sine of x is greater than the cosine of x. These are the shaded areas of the graph. I simply traced to the intersection points of these two graphs and wrote them as the domain of "x." There were four intersection points, thus resulting in two parts in the domain. → ??

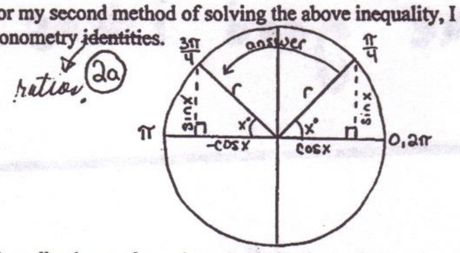
The x values of the four points were: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

The ~~domain, and the answer,~~ solution is when "x" is an element of

$\left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$.

Method Two:

For my second method of solving the above inequality, I used the unit circle and basic trigonometry ~~identities~~.



If a radius is struck on the unit circle, line r, the x value (or component, to use physics terms) equivalent to the cosine of angle x (I say, bad form making the angle value "x." Makes things rather confusing.) The y value (or component) is equivalent to the sine of angle x. The above picture, 2a, labels this. → absolute value of

Now, since we want to find where the sine of angle x is greater than or equal to the cosine of angle x, we can create a triangle with angle x = 45 degrees, since this is the first point where they are equal. We know this to be true, because for the cosine and sine to be equal, the angle x must be 45 degrees (the angle equals the inverse tangent of 1, which equals 45°.) Thus our starting point, in radians, is $\frac{\pi}{4}$. As the triangle goes counterclockwise, the sine gets larger and the cosine gets smaller, then it reverses until → absolute val they are equal again at the point, in radians, $\frac{3\pi}{4}$. This is our first interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ where the sine is greater than or equal to the cosine of the angle x.

absolute value of

– absolute value

→ once again absolute value of sin x + cos x are equal here.

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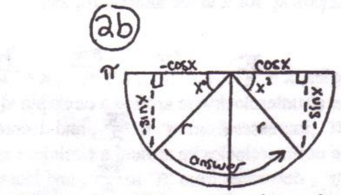
SAMPLE #1 - (con.)



Normally, there would be no other intervals, but remember, we are finding where the ABSOLUTE VALUE of sin x is greater than or equal to the ABSOLUTE VALUE of cos x. This creates another interval in the "negative" quadrants of the unit circle as shown above in picture 2b. The sine and cosine of x are again equal at the point $\frac{5\pi}{4}$. As the line r continues in its counterclockwise motion, the sine gets larger then smaller again until the point $\frac{7\pi}{4}$ where the abs sine and abs cosine are again equal. This is our second and last interval for the problem situation.

negative

The ~~domain~~ Solution for the problem is, $x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$.

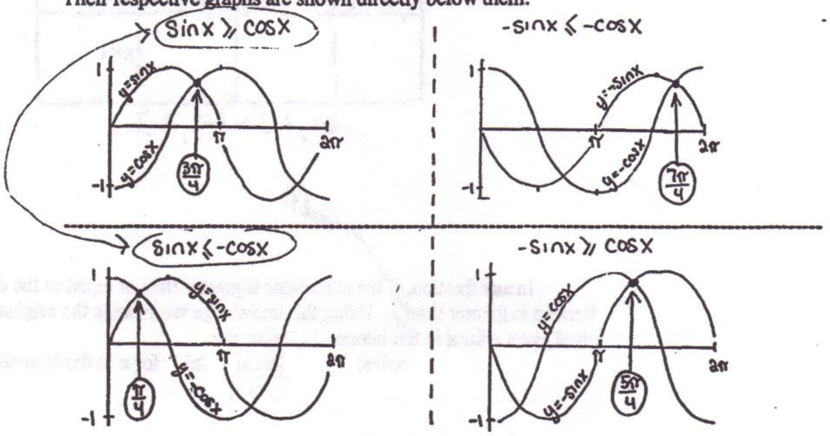
These correspond with the answers found graphically. "It's all good!"

"And now for something completely different..." -*Monty Python's Flying Circus*

Method Three:

For this method of solving the inequality, I've mixed a little algebra and a pinch of geometry. We start out with the inequality:

Algebraically, we can ~~simplify~~ decompose this inequality into the following four inequalities. Their respective graphs are shown directly below them.



Their graphs are shown in the viewing window [0, 6.2831...] by [-1, 1]. The x values are given in the problem. The y values were derived by me to best fit the problem situation. Since all these graphs are related to each other and can be combined, we can find the positive intersection points on each graph and use these to find the intervals for which |sin x| is greater than or equal to |cos x|. These points are labeled and are:

In what way?

$$\frac{3\pi}{4},\ \frac{7\pi}{4},\ \frac{\pi}{4},\ \frac{5\pi}{4}$$

When the values in the first quadrant are combined, we get the same graph as in method one. The intervals can then be seen. The domain is

$$\left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$$

Conclusions:

Since I found the same ~~domain~~ Solution (answer to the problem) using three different methods, I can only assume that this ~~domain~~ Solution must be right. My methods were sound and mathematically correct. I would like to thank all the little people who made this possible: Pythagoras, Rene Descartes, and all the "guys." This ends my writing project.

Not very humble, are we?

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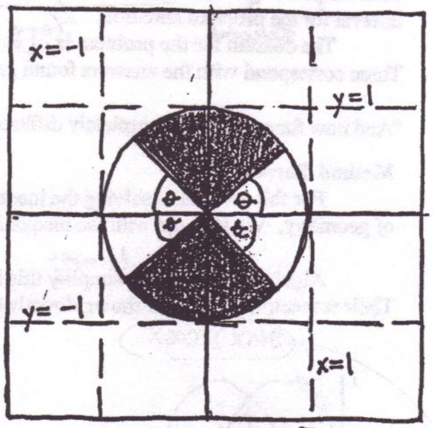
SAMPLE #2

Solve: $|\sin x| \geq |\cos x|$ for x in the interval $[0, 2\pi]$

At the angles, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, $x = \frac{7\pi}{4}$, $|\sin x| = |\cos x| = \frac{\sqrt{2}}{2}$.
As you rotate counterclockwise around a circle $|\sin x|$ increases from 0 to $\frac{\pi}{2}$, decreases from $\frac{\pi}{2}$ to π, increases from π to $\frac{3\pi}{2}$, and decreases from $\frac{3\pi}{2}$ to 2π.
As you rotate counterclockwise around a circle $|\cos x|$ decreases from 0 to $\frac{\pi}{2}$, increases from $\frac{\pi}{2}$ to π, decreases from π to $\frac{3\pi}{2}$, and increases from $\frac{3\pi}{2}$ to 2π.
Using this information we can see that where $\frac{5\pi}{4} \leq x \leq \frac{7\pi}{4}$, and where $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$, $|\sin x| \geq |\cos x|$.



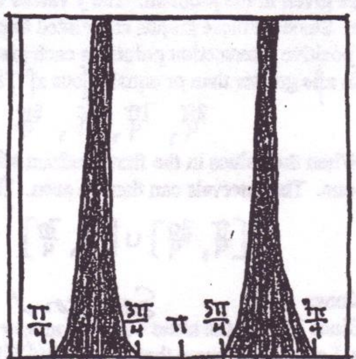
$\theta = \frac{\pi}{4}$

$[-2, 2] \times [-2, 2]$

In any fraction, if the numerator is greater than or equal to the denominator, the fraction in greater than or equal to 1. Using this knowledge we arrange the original equation of $|\sin x| \geq |\cos x|$ for x in the interval $[0,2\pi]$ to say:

solve: $\frac{|\sin x|}{|\cos x|} \geq 1$ for x in the interval $[0,2\pi]$

Using this new equation we obtain the graph:

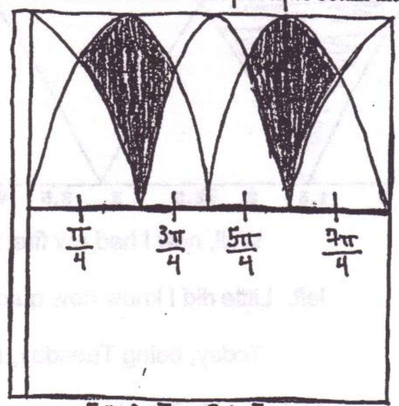


$[0, 2\pi] \times [1, 5]$

Realize that in a fraction if the numerator is greater than 0, and the denominator is equal to 0 the fraction is undefined, and that at the values $x = \frac{\pi}{2}$, and $x = \frac{3\pi}{2}$ $|\sin x| = 1$ and $|\cos x| = 0$. With this knowledge we can say by looking at the graph that $|\sin x| \geq |\cos x|$ when $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$, and when $\frac{5\pi}{4} \leq x \leq \frac{7\pi}{4}$.

SAMPLE #2 (con.)

Graphing |sin x| on the same set of axes with |cos x| we obtain the graph:



y= |sin x| (green line)
y= |cos x| (blue line)

Looking at this graph we can see that $|\sin x| \geq |\cos x|$ in the intervals $[\frac{\pi}{4}, \frac{3\pi}{4}]$ and $[\frac{5\pi}{4}, \frac{7\pi}{4}]$ for x in the interval $[0,2\pi]$.

SAMPLE # 3

My First MPI Writing Assignment

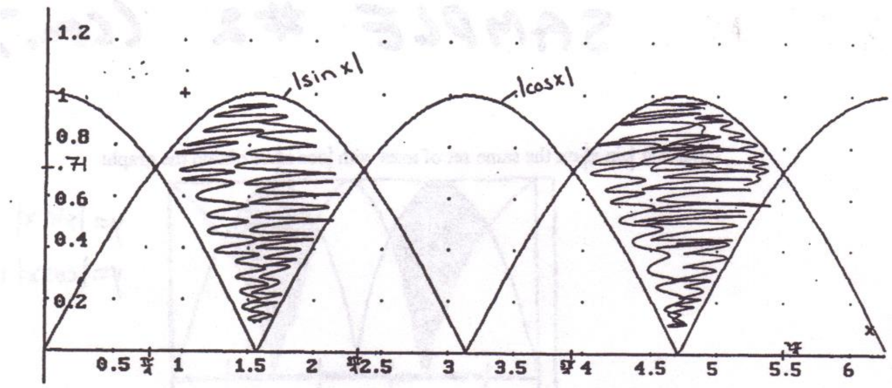
We were told to try to be creative with this paper, so I am not going to only state facts. I plan to tell the story of how I found three answers to the equation |sin x|≥ |cos x|. good.

When we first received this assignment I thought to finish it early and get it out of the way. I began to graph in class and found that if graphed separately, |sin x| and |cos x|, you could find the answer.

To observe the graph of these two, please refer to graphing page number one. The intersection points are labeled, as well as the highest y value and the lowest y value. Also, the graph only includes (0, 2π).

I know that where |sin x| was higher than |cos x| those numbers fit the original equation. These areas are shaded on the graph to show all points in that area can be correct for the problem. 50

SAMPLE #3 (con.)



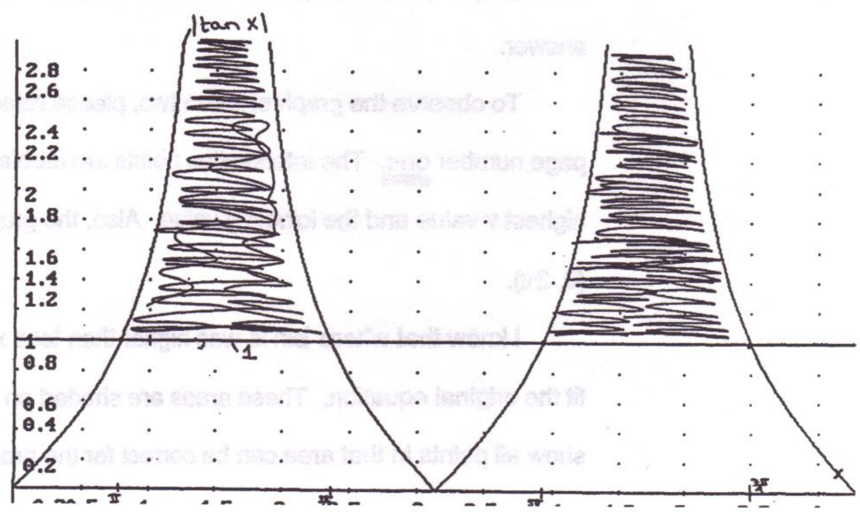
①

Well, now I had my first answer and there was still two weeks left. Little did I know how quickly two weeks would soar by.

Today, being Tuesday, I thought it would be nice to finish this assignment. Thanks to my calculator, pals and Mrs. Adams, I finally understood what to do. Although it was just graphing or solving an equation, the thought of this paper intimidated me to the point of idiocy!

I actually sat down to think in seminar today. After staring at a blank page, an idea came to me. If cos x divided to the other side, the equation became $|\sin x| / |\cos x| \geq 1$. Now it was easy. The left side of the equation equals tangent. Then I graphed $|\tan x|$ and 1. The graph I received is located on graphing page number two. All points are also noted on this graph. ← Must consider the case that $|\cos x| = 0$, check that the argument in equality still holds. −2

I traced to the intersection points and found them to be the same as on the first graph.



②

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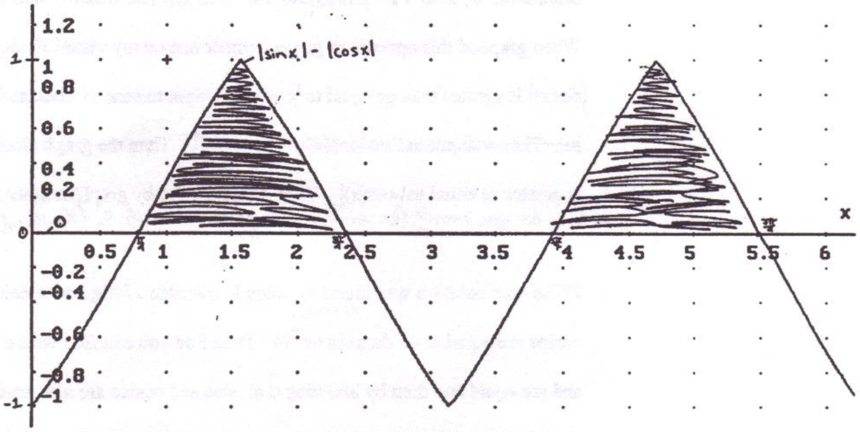
SAMPLE #3 (con.)

So I had two down and one to go. I was back to staring at the original problem. I figured that since I had algebraically moved it around to graph before, I thought I could try again.

This time I subtracted |cos x| over to the other side. The new equation became $|\sin x| - |\cos x| \geq 0$. What do ya know? It graphed separately as well! To see the third, and final graph, refer to graphing page number three.



③

The equation was answered at all points about zero, with the same intersection points as the previous two.

I finally found my three solutions. Not only did I learn the importance of algebra and graphing, I also learned to never again procrastinate a writing assignment.

I hope that you liked my story, respect my honesty and try not to grade me too harshly for it!

SAMPLE #4

[redacted]
Calculus
10-25-00

Calculus Writing Assignment

I pondered the question at hand for some time trying to find three original answers to solve the inequality of the absolute value of sine "x" is greater than or equal to the absolute value of cosine "x". I finally came up with several ways to get a solution and while they might not be the most original, they work!

1) The first solution was derived by putting each side of the inequality into my graphing calculator. I put in Y1= $|\sin(x)|$ and Y2= $|\cos(x)|$. The window was set at $[0, 2\pi]$ by $[0,1]$. When graphed this appears as graph number one on my visual sheet. Then to find where $|\sin(x)|$ is greater than or equal to $|\cos(x)|$, I ~~went to second draw and then to shade and set. Then you put in that $|\cos(x)| < Y < |\sin(x)|$~~. Then the graph shades in where $|\sin(x)|$ is greater or equal to $|\cos(x)|$. This is represented by graph number 2 on my visual sheet.

Don't give this sort of technical detail about operation of calculator.

— How do you know the endpoints of the intervals? (Answered in #2 below)

2)The next solution was found by using knowledge of trig and special triangles. Sine and cosine are equal at 45 degrees or $\pi/4$. Therefore you can find where they first intersect and are equal and then by knowing that sine and cosine are also equal at $3\pi/4$ you can find where they intersect again. You also know by using the unit circle that they are equal at $5\pi/4$ and $7\pi/4$. Then you know that between these two points $|\sin(x)|$ is greater than or equal to the $|\cos(x)|$. See graph number three and four on the visual sheet.

3) The final solution I came up with was that I manipulated the equation algebraically. I divided both sides by $|\cos(x)|$ ~~off from $|\sin(x)|$~~ and then had $|\tan(x)|$ is greater than or equal to one. Then I graphed both sides of the equation. Where the graph of $|\tan(x)|$ and one cross is

This assumes $|\cos(x)| \neq 0$. If it is, you must show to us why the inequality (trivially) still holds.

—2

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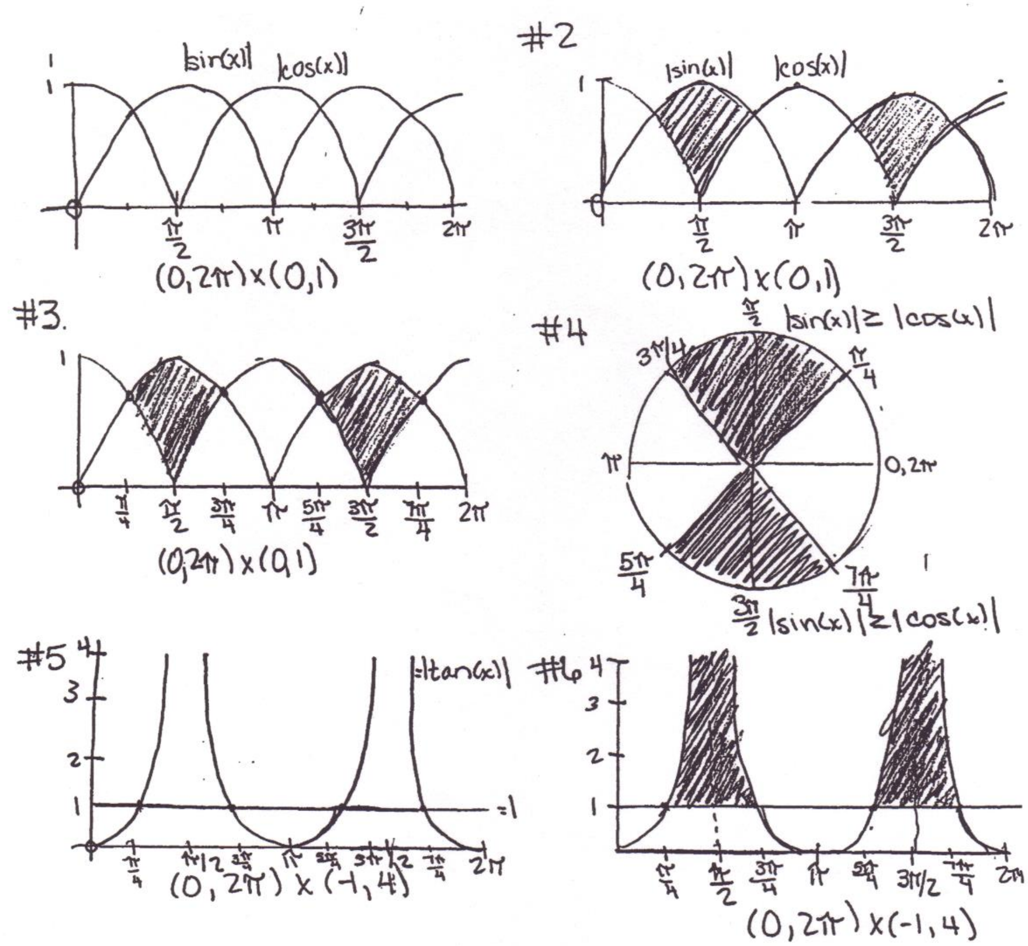
SAMPLE #4 (con)

where |sin(x)| and |cos(x)| are equal which is at π/4 and 3π/4 and then at 5π/4 and 7π/4. This is shown in graph number five on my visual sheet. Then I shaded the parts where |tan(x)| is greater than or equal to one and discovered that there are vertical asymptotes at π/2 and 3π/2. See graph number six for this solution.

not at all!

Well to be honest these solutions are all kind of boring and unoriginal but they all get you to the same answer and they are all mathematically correct (hopefully?) so after these different solutions, I have come to realize that there are many ways of solving one inequality and there is more to calculus than just the right answer.

Visual Sheet — excellent!



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