

# A video Calculus I course

Recorded at the University of Missouri – Kansas City in 2005. All times are approximate. About **29 hours total**.

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**YouTube Playlist:**                      **Course: Calculus I with Professor Richard Delaware**

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## Lecture 1    46:50

### UNIT 0 - FUNCTIONS: A Review of Precalculus

#### Beginning

- Definition of a Function [7.5 min.]
- Visualizing Functions: Graphs [10.5 min.]
- Domain (& Range) of Functions [7.5 min.]
- Some Exercises [11 min.]

#### Graphing Technology

- Viewing Windows [3.5 min.]
- Zooming In or Out [2.5 min.]
- Errors in Resolution [5.5 min.]

## Lecture 2    36:29

### New Functions From Old

- Operations on Functions [10 min.]
- How Operations Affect Function Graphs [6.5 min.]
- Functions with Symmetric Graphs [2 min.]
- Some Exercises [8 min.]

### Families of Functions

- The Power Function Family  $y = x^p$  [7 min.]
- The Polynomial Function, and Rational Function Families [4 min.]

## Lecture 3 1:12:41

### Trigonometry for Calculus

- Right Triangle Trigonometry [11.5 min.]
- Trigonometric Graphs [5 min.]
- Handy Trigonometric Identities [5 min.]
- Laws of Sine and Cosine [4 min.]
- Trigonometric Families [4 min.]

### Inverse Functions

- A Function Inverse to Another Function [6 min.]
- When do Inverse Functions (& Their Graphs) Exist? [4 min.]
- Inverse Trigonometric Functions [10 min.]

### Exponential & Logarithmic Functions

- The Exponential Function Family [6 min.]
- The Logarithmic Function Family [6.5 min.]
- Solving Exponential & Logarithmic Equations [6 min.]

## Lecture 4 1:32:11

### UNIT 1 - LIMITS of Functions: Approach & Destination

#### Intuitive Beginning

- A New Tool: The "Limit" [24 min.]
- Some Limit Examples [6.5 min.]
- Two-sided & One-sided Limits [17 min.]
- Limits that Fail to Exist: When  $f(x)$  grows without bound [9 min.]
- Limits at Infinity: When  $x$  grows without bound [11.5 min.]
- More Limits that Fail to Exist: Infinity & Infinite Indecision [6 min.]
- An Exercise on Limits [5 min.]

**Lectures 5-6 2:21:54****The Algebra of Limits as  $x \rightarrow a$** 

- Basic Limits [7.5 min.]
- Limits of Sums, Differences, Products, Quotients, & Roots [11 min.]
- Limits of Polynomial Functions [7 min.]
- Limits of Rational Functions & the Apparent Appearance of  $0/0$  [17 min.]
- Limits of Piecewise-Defined Functions: When One-sided Limits Matter! [12 min.]
- Some Exercises [11 min.]

**The Algebra of Limits as  $x \rightarrow \pm \infty$ : End Behavior**

- Basic Limits [4.5 min.]
- Limits of Sums, Differences, Products, Quotients, & Roots [10 min.]
- Limits of Polynomial Functions: Two End Behaviors [9 min.]
- Limits of Rational Functions: Three Types of End Behavior [16 min.]
- Limits of Functions with Radicals [9.5 min.]
- Some Exercises [6.5 min.]
- Limits of  $\ln(x)$ ,  $e^x$ , and More [7 min.]

**Lecture 7 1:03:51****Continuous Functions**

- Functions Continuous (or not!) at a Single Point  $x=c$  [14.5 min.]
- Functions Continuous on an Interval [6 min.]
- Properties & Combinations of Continuous Functions [14 min.]
- The Intermediate Value Theorem & Approximating Roots:  $f(x) = 0$  [13 min.]
- Some Exercises [13.5 min.]

**Lecture 8 31:00****Trigonometric Functions**

- The 6 Trigonometric Functions: Continuous on Their Domains [4 min.]
- When Inverses are Continuous [3 min.]
- Finding a Limit by "Squeezing" [6 min.]
- $\sin(x)/x \rightarrow 1$  as  $x \rightarrow 0$ , and Other Limit Tales [11 min.]
- Some Exercises [7 min.]

## Lecture 9 54:33

### UNIT 2 - The DERIVATIVE of a Function

#### Measuring Rates of Change

- Slopes of Tangent Lines [14 min.]
- One-Dimensional Motion [7 min.]
- Average Velocity [6.5 min.]
- Instantaneous Velocity [8 min.]
- General Rates of Change [8.5 min.]
- Some Exercises [10.5 min.]

## Lecture 10 52:16

#### What is a Derivative?

- Definition of the Derived Function: The "Derivative", & Slopes of Tangent Lines [11 min.]
- Instantaneous Velocity [2 min.]
- Functions Differentiable (or not!) at a Single Point [9 min.]
- Functions Differentiable on an Interval [3 min.]
- A Function Differentiable at a point is Continuous at that point [6 min.]
- Other Derivative Notations [4 min.]
- Some Exercises [11 min.]

## Lecture 11 1:14:22

#### Finding Derivatives I: Basic Rules

- The Power Rule [13.5 min.]
- Constant Multiple, Sum, & Difference Rules [5.5 min.]
- Notation for Derivatives of Derivatives [6 min.]
- Some Exercises [7 min.]

#### Finding Derivatives II:

- The Product Rule [14.5 min.]
- The Quotient Rule [9 min.]
- Some Exercises [10 min.]

## Lecture 12 53:00

#### Finding Derivatives III:

- The Sine Function [5 min.]
- The Other Trigonometric Functions [9 min.]
- Some Applications [11 min.]

#### Finding Derivatives IV:

- The Chain Rule: Derivatives of Compositions of Functions [14.5 min.]
- Generalized Derivative Formulas [5.5 min.]
- Some Exercises [8 min.]

## Lecture 13 48:13

### When Rates of Change are Related

- Differentiating Equations to "Relate Rates" [10 min.]
- A Strategy [14.5 min.]
- An Exercise [11 min.]

### More on Derivatives

- Local Linear Approximations of Non-Linear Functions [7 min.]
- Defining " $dx$ " and " $dy$ " Alone [5 min.]

## Lecture 14 55:56

### UNIT 3 - Some Special DERIVATIVES

#### Implicit Differentiation

- Functions Defined Implicitly [7 min.]
- Derivatives of Functions Defined Implicitly [7.5 min.]
- The Derivative of Rational Powers of  $x$  [6 min.]
- Some Exercises [6 min.]

#### Derivatives Involving Logarithms

- Derivatives of Logarithmic Functions [14.5 min.]
- The "Logarithmic Differentiation" Technique [5.5 min.]
- The Derivative of Irrational Powers of  $x$  [3.5 min.]
- Some Exercises [5 min.]

## Lecture 15 1:11:43

#### Derivatives Involving Inverses

- Derivatives of Inverse Functions [10 min.]
- Derivatives of Exponential Functions [7.5 min.]
- Derivatives of Inverse Trigonometric Functions [6 min.]
- Some Exercises [5 min.]

#### Finding Limits Using Differentiation

- Limits of Quotients that appear to be "Indeterminate": The Rule of L'Hopital [14 min.]
- Some Examples [11 min.]
- Finding Other "Indeterminate" Limits [16 min.]

**Lecture 16 1:06:37**

**UNIT 4 - The DERIVATIVE Applied**

**Analyzing the Graphs of Functions I**

- Increasing & Decreasing Functions: The 1<sup>st</sup> Derivative Applied [16 min.]
- Functions Concave Up or Concave Down: The 2<sup>nd</sup> Derivative Applied [14 min.]
- When Concavity Changes: Inflection Points [19 min.]
- Logistic Growth Curves: A Brief Look [3 min.]
- Some Exercises [14 min.]

**Lecture 17 1:01:11**

**Analyzing the Graphs of Functions II**

- Local Maximums & Minimums [11 min.]
- The 1<sup>st</sup> Derivative Test for Local Maximums & Minimums [7.5 min.]
- The 2<sup>nd</sup> Derivative Test for Local Maximums & Minimums [9.5 min.]
- Polynomial Function Graphs [15 min.]
- Some Exercises [17.5 min.]

**Lecture 18 56:40**

**Analyzing the Graphs of Functions III**

- What to Look For in a Graph [2 min.]
- Rational Function Graphs [19.5 min.]
- Functions Whose Graphs have Vertical Tangents or Cusps [16 min.]
- Some Exercises [18.5 min.]

**Lecture 19 44:30**

**Analyzing the Graphs of Functions IV**

- Global Maximums & Minimums [4.5 min.]
- Global Extrema on (finite) Closed Intervals [10 min.]
- Global Extrema on (finite or infinite) Open Intervals [13 min.]
- When a Single Local Extremum must be Global [4.5 min.]
- Some Exercises [12 min.]

**Lecture 20 1:02:23**

**Optimization Problems**

- Applied Maximum & Minimum Problems [4 min.]
- Optimization over a (finite) Closed Interval:  
Maximizing Area or Volume, Minimizing Cost [21.5 min.]
- Optimization over Other Intervals: Minimizing Materials or Distance [11 min.]
- An Economics Application: Cost, Revenue, Profit, & Marginal Analysis [9 min.]
- Some Exercises [17 min.]

## Lecture 21 1:05:46

### Newton's Method for Approximating Roots of Equations

- Development of the Method [12 min.]
- Strength & Weaknesses of the Method [5.5 min.]

### The Mean Value Theorem for Derivatives

- A Special Case of the Mean Value Theorem: Rolle's Theorem [8.5 min.]
- The (Full) Mean Value Theorem for Derivatives [20 min.]
- Direct Consequences of This Mean Value Theorem [13 min.]
- Some Exercises [7 min.]

## Lecture 22 36:30

### One-Dimensional Motion & the Derivative

- Rectilinear Motion Revisited [4.5 min.]
- Velocity, Speed, & Acceleration [12 min.]
- Analyzing a Position Graph [8.5 min.]
- An Exercise [11.5 min.]

## UNIT 5 - The INTEGRAL of a Function

### The Question of Area

- Brief History and Overview [17.5 min.]

## Lecture 23 1:05:51

### The Indefinite Integral

- "Undo-ing" a Derivative: Antiderivative = Indefinite Integral [16 min.]
- Finding Antiderivatives [22 min.]
- The Graphs of Antiderivatives: Integral Curves & the Slope Field Approximation [16.5 min.]
- The Antiderivative as Solution of a Differential Equation [5 min.]
- Some Exercises [6.5 min.]

## Lecture 24 36:02

### Indefinite Integration by Substitution

- The Substitution Method of Indefinite Integration: A Major Technique [7.5 min.]
- Straightforward Substitutions [10.5 min.]
- More Interesting Substitutions [11.5 min.]
- Some Exercises [7 min.]

## Lecture 25 50:09

### Area Defined as a Limit

- The Sigma Shorthand for Sums [13.5 min.]
- Summation Properties & Handy Formulas [9 min.]
- Definition of Area "Under a Curve" [15 min.]
- Net "Area" [4 min.]
- Approximating Area Numerically [2.5 min.]
- Some Exercises [6.5 min.]

## Lecture 26 40:41

### The Definite Integral

- The Definite Integral Defined [11.5 min.]
- The Definite Integral of a Continuous Function = Net "Area" Under a Curve [6 min.]
- Finding Definite Integrals [10.5 min.]
- A Note on the Definite Integral of a Discontinuous Function [6 min.]
- Some Exercises [6.5 min.]

## Lecture 27 45:49

### The Fundamental Theorem of Calculus

- The Fundamental Theorem of Calculus, Part 1 [15 min.]
- Definite & Indefinite Integrals Related [7.5 min.]
- The Mean Value Theorem for Integrals [9.5 min.]
- The Fundamental Theorem of Calculus, Part 2 [7 min.]
- Differentiation & Integration are Inverse Processes [2 min.]
- Some Exercises [5 min.]

## Lecture 28 52:41

### One-Dimensional Motion & the Integral

- Position, Velocity, Distance, & Displacement [16 min.]
- Uniformly Accelerated Motion [12 min.]
- The Free Fall Motion Model [6.5 min.]
- An Exercise [5 min.]

### Definite Integration by Substitution

- Extending the Substitution Method of Integration to Definite Integrals [9 min.]
- Some Exercises [4 min.]

## Lecture 29 23:57

### UNIT 6 - The DEFINITE INTEGRAL Applied

#### Plane Area

- Area Between Two Curves [One Floor, One Ceiling] [11 min.]
- Area Between Two Curves [One Left, One Right] [7.5 min.]
- An Exercise [5.5 min.]

## Lecture 29 Part II 49:06

#### Volumes I

- Volumes by Slicing [12.5 min.]
- Volumes of Solids of Revolution: Disks [15.5 min.]
- Volumes of Solids of Revolution: Washers [12 min.]
- Some Exercises [8.5 min.]

## Lecture 30 20:38

#### Volumes II

- Volumes of Solids of Revolution: Cylindrical Shells [14.5 min.]
- An Exercise [6 min.]

#### Length of a Plane Curve

- Finding Arc Lengths [11.5 min.]
- Finding Arc Lengths of Parametric Curves [6.5 min.]

## Lecture 31 44:01

#### Average Value of a Function

- Average (Mean) Value of a Continuous Function [13 min.]

#### Work

- Work Done by a Constant Force [3 min.]
- Work Done by a Variable Force [13.5 min.]
- Do-It-Yourself Integrals: Pumping Fluids [8 min.]
- Work as Change in Kinetic Energy [6 min.]
- An Exercise [5 min.]