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# One Golden Age

*Mathematics Since 1801*

ERIC T. BELL

THE year 1942 seems scarcely an appropriate time to mention a golden age in anything. Civilization admittedly is in a bad way, and the pessimism induced by this obvious fact seems to have carried over to the arts and literature. All the great pictures were painted centuries ago; no deathless novel has joined the classics on the top shelf in sixty years; current poetry and music are worthless; philosophy has been duller than mutton for God knows how long; religion is not what it was in the days of Chatauqua; science has gone insane, and most of us with it; western civilization is senile and presently will be dead and damned. It is all very depressing—if you are gullible enough to believe much of it.

But granting that there may be something in the Egyptologist Petrie's cyclic theory of cultural epochs, we may agree that certain forms of art reach a maximum of excellence, maintain their perfection for a brief season, and then decline in a sterile imitativeness of the highest, to fritter out in meaningless refinements and collapse at last in total imbecility. We have been assured, for example, that the Greeks did better in the plastic arts than any of their successors; that the Elizabethans made it superfluous for anyone after them to write a love lyric; and that the Victorians did things with horsehair and crimson plush that will never be surpassed.

Intone or snuffle the roll of great names, and enjoy a good wet cry for the plight of your own age: Homer, Sappho, Aeschylus; Socrates, Plato, Kant; Dante, Shakespeare, Goethe; Alfred Tennyson, William Morris, Martin Tupper—all these only from the immortals in literature or philosophy or interior decorating, and all as dead as Pompey. Take any list you like of the hundred best symphonies, or the thousand best pictures, or the ten thousand best books of all time—or at least of as much of all time as has yet elapsed —and you will feel like jumping off the roof. And for anything a mere spectator of literature and the arts can say, you may be doing the wisest thing, unless—

Unless what? Unless you happen to have noticed that not a single name in science has been called. True, Goethe rated himself a scientist of the first rank, and regarded all his contribution to literature as a trifle of little account. Confident that he would be remembered for his physics when his *Faust* had been forgotten, he slaved to set Isaac Newton right on optics. But beyond convincing the intelligentsia of his home town that Newton was a boor in ethics and a charlatan in science, the poet did not accomplish much of interest to scientists. The "culture vultures" in Goethe's day were like that—as witless scientifically as snollygosters. Some of them may even have been unaware that they were living in a notable age of poetry. Certainly none of them suspected that the golden age of the oldest of all the sciences had dawned in the year 1801. How could they, when their idol of the moment was living in the Stone Age so far as mathematics was concerned, and had never even heard the name of that unobtrusive fellow-countryman of his, who with Archimedes (287-212 B.C.) and Newton (1642-1727) is universally considered one of the three greatest mathematicians in history? Goethe's *Faust* (first part) appeared in 1808, the *Disquisitiones Arithmeticae* of Gauss (1777-1855) in 1801.

The master works of Archimedes are still as vital in mathematics and science as they were twenty centuries ago, though only a few highly specialized historians ever read a word that Archimedes wrote. *Faust* will almost certainly outlive Goethe's fatuities in optics, if it has not already done so. But to extrapolate from the past to the future, it seems unlikely that anything in *Faust* will have become an ingrained habit of thought for rational men two thousand years hence. Erudite scholars of that distant day will no doubt be able to make some sense of the quaint antics of Mephistopheles, and specialists in the science of the soul may at last have discovered what ailed Gretchen. But unless everybody then is two thousand years stupider than the majority of us are today, only inmates of hospitals for the mentally ill will be seriously affected by anything in *Faust*.

What of the *Disquisitiones?* Will anybody be reading it then? Probably not. Only one professional mathematician in thousands today has ever glanced at a line of it. Yet the thoughts of all have been influenced in some degree by what Gauss wrote in 1801, for it hinted at a liberation of the imagination in all mathematics. More than that, it was the masterpiece of this greatest mathematician since Newton, and it announced the sudden dawn of the golden age of mathematics.

Archimedes marked one great age in mathematics, and he still lives. Newton marked another, and the course of his life will be run only when our civilization ends. Gauss heralded the greatest of all, and as long as human beings retain their capacity for rational thought in science, or in mathematics, or in philosophy, the surge of discovery which he initiated will echo in their thoughts. All this, of course, is on the supposition that Gauss shares the historical fate of Archimedes. He may not. Our successors may find that superstition suits them better than science, and insist on going to perdition with Faust. Well, we can't stop them if that is what they are to want. May they have a better time at it than we are having in our strenuous attempt to anticipate them. Possibly they will; for like Goethe's hero they may welsh on their bet with the devil and be saved by their good works. In passing, it may be significant that the English Marlowe's Dr. Faustus paid his bet and was damned like a gentleman.

## II

The golden age of mathematics that began with Gauss in 1801 continued to 1939, though naturally the effects of the first world war were plainly apparent by 1930. French mathematics, for example, was then at a lower ebb than at any previous time since the mid-eighteenth century. The old leaders were through, and the men of middle age who normally might have succeeded them either had been killed in

1914-1918 or had drifted away from mathematics. In Germany a similar but less marked decline was evident. The general level remained high. In the United States, Russia, and Japan there was an increase in mathematical productivity. Without undue chauvinism it may be said that American mathematics was not inferior in either quantity or quality during this period to the general output of Europe. The net outcome was a slight continued acceleration in the rate of progress. Then came the second world war, which may halt progress and end one of the world's great ages of intellectual achievement.

## III

To dispose here of a preposterous fable that has recently gained currency through the highly ingenious but singularly misinformed speculations of an eminent sociologist, mathematics did not, as he asserts, reach its peak about the middle of the eighteenth century. This item of misinformation is exhibited—wherever science is discussed—to prove that all our culture is decadent and our civilization so rotten that only the retarded chemistry of its corruption keeps it from falling apart. So far as mathematics is concerned, such a doleful conclusion could be reached only by an evangelical humanist bent on scaring the sin out of us by prematurely preaching the end of the world.

No mathematically literate man agrees with the learned sociologist's conclusion, for any such man knows that it does not rise to the lowest level of the ridiculous, but is merely idiotic. He knows also that the nineteenth century alone created approximately five times as much new mathematics as had been produced in the whole of preceding history. And he knows further that the first four decades of the twentieth century showed no over-all diminution in the output.

How is it possible, it may be asked, for a conscientious and phenomenally industrious scholar to go so egregiously astray on a matter of verifiable fact? Suppose you wish to follow in this eminent man's footsteps; what must you do? It is quite simple! Base your conclusions on a scandalously naive though volumetrically imposing catalogue of all the advances of the past six thousand years in all the sciences, after making sure that the cataloguer has used only the broom-and-dustpan technique in collecting his items; employ no sweeper who has looked at any mathematics more recent than the early eighteenth century; and last, resolutely resist the temptation to learn any mathematics that was not already antiquated a hundred years before you were born. This procedure will infallibly lead you to the most remarkable conclusions—if that is your goal—all of which are at variance with ascertainable fact. But your labors will not pass unnoticed or unrewarded. You may even get a couple of honorary degrees for your outstanding services in the propagation of rubbish and for your call to repentance—"the end of the world is at hand."

## IV

A mere list of the achievements of mathematics since 1801 would recall the story to those already familiar with it, but might be so much hieroglyphics to others. All that can be attempted here is a description of the principal characteristic which distinguishes mathematics since 1801 from all, or nearly all, that preceded it. In a word it is *generality.*

Greek mathematics, it has often been observed, resembles Greek art. Each theorem is a distinct creation, perfect in itself and distinguished from all others by its own completeness. Though proved by means of other theorems, each is its own end, and its simple beauty the justification of its existence. It is like a white temple on a bare hilltop, alone and an embodiment of perfection, suggesting nothing further to be done. It is eternally static and eternally lifeless.

Though it may seem paradoxical to lovers of Greek epic and tragedy, the Greek mind in mathematics was narrowly limited, with an ineradicable predilection for the finite and strictly bounded. Its primitive horror of the infinite—the unlimited, the endless, the unbounded — was permanently confirmed by an unfortunate experience in the fifth century B.C., when apparently sound logic applied to the analysis of motion produced certain paradoxes the Greek geometers were unable to resolve.

Any usable description of motion involves the concept of continuously varying *number.* For motion is based on a postulated *infinite divisibility* of *space* into *points,* and of *time* into *instants.* It was in this connection that the paradoxes arose. Terrified that all reasoning about the infinite might be infected with subtle paradox, the Greek mathematicians abandoned the attempt to create a dynamic theory of *number,* or a usable *mathematics of continuity,* and halted abruptly on the very threshold of modern mathematics. They thus missed discovering the simple general principles which would have given them any number of individual theorems they might desire by uniformly applicable processes. Discouraged, they returned to their familiar tools and continued to chisel out disconnected objects of art. Even Archimedes, who used anything that came to hand in making some of his most spectacular discoveries, recast his creations in the rigid classic mold before sharing them with his contemporaries.

If Greek mathematics needs an epitaph, the following might be not too inappropriate: Here lies the body of a sterile perfection, the victim of its devotion to a barren purity.

## V

In the next great age of mathematics, that inaugurated by Newton and Leibniz in the Seventeenth Century, mathematics escaped from the cramping bonds of the finite. The differential and integral calculus of these two men provided the long-sought solution of the problem of accurately describing motion, and made possible a rational analysis of all continuous change. Mathematics became dynamic.

The ceaseless flux of nature and the phenomena of growth and decay were now within the grasp of exact reasoning. The ancient horror of the infinite was dispelled forever in the brilliant achievements of the new mathematics in geometry, in astronomy, and in the physical sciences. An abrupt transition from the *exclusive special* to the *inclusive general* was already evident in the newer type of problem attacked in geometry. Instead of minutely anatomizing the geometry of this or that particular curve or surface, mathematicians began the classification of all imaginable curves and surfaces, and sought properties common to all the members of whole classes of geometrical objects. This, however, was only a shadow of what was to come in the nineteenth century.

Throughout the eighteenth century the ablest men devoted their strongest efforts to developing the Newtonian theory of gravitation and the application of the calculus to the physical sciences. Consequently the innate freedom of mathematics itself was not envisaged. Preoccupied as they were with the innumerable uses of the new mathematics as an implement of discovery in the sciences, the leading mathematicians of the age paid scant attention to the marvellous instrument of power in their hands, and were content merely with the beautiful things it turned out in overwhelming profusion. Its own infinite potentialities of further development were not even suspected. But the decisive transformation had occurred, though unnoticed. Mathematics was as free as the human mind. Instead of the frigidly perfect Greek temple, mathematics was now a Gothic cathedral with no suggestion of a cramped finality in any line or spire of it.

## VI

At last, with the opening of the nineteenth century, attention turned to the development of mathematics on its own account. Realizing that the fatedness, the eternal necessity of any particular science of *space* or *number* is an illusion, Gauss as a boy of twelve had imagined the possibility of geometries other than the system of the Greeks. That ancient geometry had stood alone for over two thousand years. By the early 1830's Gauss and others had elaborated the first of the non-Euclidean geometries. Thirty years more, and the creation of new geometries for scientific or esthetic reasons had become almost a trade or a pastime.

It was the same in the higher arithmetic and in algebra, in the theory of functions and in the outgrowths of the calculus, in mathematical logic and in the study of abstract systems of relations between things of any kind whatever. Everywhere there was free inventiveness subject only to avoidance of self-contradiction. When any man of normal intelligence with some training and a little imagination could invent a new kind of algebra or a novel geometry, individual theorems as ends in themselves lost their attractiveness. General principles and powerful methods, each capable of producing an endless variety of theorems, were sought after, found, and exploited only long enough to certify their universality in their respective domains, or to satisfy their inventors' need for something less abstract than complete generality, when they were put aside for possible future use.

Simultaneously, numerous new subjects, whose aim and content would have been unimaginable to even the greatest mathematicians of the past, came into being, served the purposes for which they had been created, and passed into the rapidly increasing stock of positive knowledge. Though possible scientific use for these things was but seldom the occasion of their creation, they frequently justified their existence in the eyes of scientists and other practical men by proving to be the appropriate machinery for application to some new and rapidly developing science or technology. The mind had anticipated the hand.

And so it continued to the first day of the second world war.