MATH 402 -- ADVANCED ANALYSIS I (3)
Professor: Richard Delaware

Course Web Page: http://d.web.umkc.edu/delawarer/RDanalysis1.htm
Department Web Site: http://cas.umkc.edu/mathematics/

FALL 2014    MW 4:00 - 5:15 pm    Haag Hall Room 302

• The prerequisites for this course are Math 250 (Calculus III) and Math 301 (On Solid Ground: Sets and Proof).

TEXTS:  [This course is the designated Capstone course for Mathematics majors.]
Required: ELEMENTARY REAL ANALYSIS, B. S. Thomson, A. M. Bruckner, J. B. Bruckner, 2nd Ed.
Recommended: HOW TO READ AND DO PROOFS, Daniel Solow (paperback).
• In the primary text, we will cover most of Chapters 1-5, 7, and parts of 8. This text is available as a free PDF file, or a print-on-demand copy. See information at: http://cas.umkc.edu/mathematics/Math402Text.htm.

EXAMS:
• There will be three 75 min. Exams (100 points each) during the semester, each given a letter grade.
• There may be some announced 20 min. Quizzes.
• The FINAL EXAM (150 points) will be held in our usual classroom, 8:00-10:00 am, Fri. Dec. 19.

HOMEWORK:
• A list of suggested homework problems will be posted. Selected problems will be collected and graded.
• For homework use only one side of standard 8½ x 11 paper. When due, put it on my desk at the start of class.
• Late assignments of any kind are NOT accepted, except by prior arrangement.

CONTACT:
• I will contact you through your official UMKC email address.
• My Office: Manheim Hall 306 A. Office Hours: M 2:30-3:30, T 3-3:50, W 11-11:50 & 2:30-3:30, R 3-3:50. I can also be available otherwise by appointment on M-R only. I have a mailbox in the Math Dept office, Haag Hall 206. Do NOT slip work under my office door (it can get lost).
• E-mail address: delawarer@umkc.edu. This is the best way to reach me.
• My Phone: (816) 235-2850. Do NOT call the Department Office to leave me messages.

GRADES:
• 3 Exams (100 pts ea.) and Final Exam (150 pts)   70 %
• Homework and Quizzes                          30 %
• At the end of the semester I will drop your "most damaging" score. However, the Final Exam cannot be dropped.

Please refer to the following web page and the linked resources for critical information regarding course policies and resources. You are expected to abide by all the rules and regulations regarding student conduct referenced in these pages: http://cas.umkc.edu/CPR/
STUDENT LEARNING OUTCOMES – Math 402:

Properties of the Real Numbers
- The Real Number system – Algebraic (Field) Structure (9 axioms), Order Structure (4 axioms), and Completeness Axiom
- Upper/Lower bounds, Maximum/Minimum, Supremum/Infimum
- The Real Number system – Archimedean property; Inductive property of N
- Dense sets; the Rational numbers are dense in R
- The Real Number system – Metric Structure (absolute value, distance)

Sequences
- Sequences as functions from N to R
- Examples of sequences (arithmetic, geometric, iterative, partial sums)
- Countable sets, open intervals (hence R) are not countable (Cantor’s theorem)
- Limit of sequences (convergence; epsilon, n), uniqueness of limits
- Limit of sequences (divergence to infinity)
- Bounded sequences; Every convergent sequence is bounded
- Algebra of limits of sequences (multiples, sums/differences, products, quotients)
- Order properties of limits of sequences (≤ theorem, squeeze theorem, absolute value, max/mins)
- Increasing, decreasing, non-decreasing, non-increasing sequences; Monotone Convergence theorem
- Important examples of limits of sequences (geometric, roots, sums of geometric, decimals, exp(x))
- Subsequences; every sequence contains a monotone subsequence; Bolzano-Weierstrass theorem (every bounded sequence contains a convergent subsequence)
- Cauchy criterion for convergence of a sequence
- Limit superior, limit inferior, basic theorems

Infinite Sums
- Finite sums, notation, telescoping sums, partial sum of geometric sequence
- Series, convergence/divergence, basic algebra properties of convergent series
- Important examples of convergent series (telescoping series, geometric series, harmonic series, alternating harmonic series, p-harmonic series
- Boundedness criterion for convergence of a series, and Cauchy criterion for convergence of a series
- Absolutely convergent series, absolute convergence theorem, non-absolutely convergent series
- Series Convergence Tests:
  - Trivial test
  - Direct Comparison tests I and II
  - Limit Comparison tests I and II
  - Ratio Comparison test
  - D’Alembert’s Ratio test
  - Cauchy’s Root test
  - Integral test
  - Alternating Series test
- Rearrangements of series, unconditional and conditional convergence
- Dirichlet and Riemann’s theorems that a series is unconditionally convergent iff absolutely convergent

Sets of Real Numbers
- Intervals, neighborhoods, interior points, isolated points, points of accumulation, boundary points
- Closed sets, closure of a set, open sets, interior of a set
- Every nonempty open set is the countable union of disjoint open intervals
- A set is open iff its complement is closed; properties (unions, intersections, etc.) of open and closed sets
- An illustrative example: Local boundedness of functions; Theorem to be proved in several ways below: A function locally bounded on a closed and bounded set is bounded on that set.
- Compactness arguments: Bolzano-Weierstrass property, Cantor’s Intersection property and theorem, Cousin’s Lemma, definitions of full and open covers, Heine-Borel property and theorem, Compact sets defined
- Countable sets revisited
Continuous Functions

- Definitions of the limit of a function (epsilon-delta definition, sequence definition, mapping [open sets] definition)
- One-sided limits of functions, infinite limits of functions
- Function Limit properties (uniqueness, boundedness, boundedness away from zero)
- Algebra of limits of functions (multiples, sums/differences, products, quotients)
- Order properties of limits of functions ($\leq$ theorem, squeeze theorem, absolute value, max/mins)
- Composition of functions and limits
- Important examples of functions (polynomials, rational functions, exponential functions, characteristic function of the rationals, Dirichlet function, non-decreasing functions with jumps, step functions)
- Distance of a closed set to a point, Characteristic function of the Cantor set, limits superior and inferior
- Continuity: Intermediate Value Property, continuity at a point in a neighborhood, at endpoints, and on an arbitrary set
- Four equivalent definitions of continuity at a point of an arbitrary set:
  - Epsilon-delta version
  - Limit version
  - Neighborhood (open set) version
  - Sequence version
- A function is continuous on a set iff the inverse image of every open set is open.
- Properties of continuous functions (multiples, sums, products, quotients, polynomials, rational functions, compositions)
- Uniform continuity; a function uniformly continuous on a bounded interval is bounded; a function continuous on $[a, b]$ is uniformly continuous (and hence bounded)
- A function continuous on a closed and bounded set possesses a global maximum and global minimum
- Darboux property of continuous functions
- Points of discontinuity (removable, jump, essential)
- Monotonic functions; have only jump discontinuities; the set of points of discontinuity of a function monotone on $[a, b]$ is countable, so the function is continuous on a dense set of points in $[a, b]$.
- For any real function on $[a, b]$ the set of points at which it has a removable discontinuity and at which it has a jump discontinuity are both countable

Differentiation

- Difference quotient, definition of derivative at a point, differentiability at a point implies continuity at that point
- Computation of derivatives (all algebraic rules, chain rule, power rule)
- Local extrema theorem, Rolle’s theorem, mean value theorem for derivatives, differentiability and monotonicity theorem
- The derivative has the Darboux property; derivative of an inverse function
- [L’Hopital’s rule mentioned if there’s time]

The Integral

- Cauchy’s theorem on the definition of the definite integral of a continuous function on $[a, b]$
- Properties of the integral of a continuous function (additive, linear, monotone, absolute value)
- Fundamental theorem of calculus in two parts for continuous functions
- Definition of improper integrals
- The Riemann integral; all continuous functions are Riemann integrable; all Riemann integrable functions are bounded
- Quickly, the Riemann-Lebesgue theorem: A function on $[a, b]$ is Riemann integrable iff the function is bounded and its set of points of discontinuity is of measure zero.
- Examples of classes of Riemann integrable functions
- Properties of the Riemann integral (additive, linear, monotone, absolute value)
- Fundamental theorem of calculus in two parts for Riemann integrable functions
- [Improper Riemann integral mentioned if there’s time]