MPI

The Mathematics and Physics Institute NEWSLETTER

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OPEN HOUSE -- NOV. 8, 1987

!COME!

Sunday Nov. 8, from 2-4 pm, the MPI will hold its annual OPEN HOUSE for parents, teachers, counselors, administrators, and anyone interested in talking to the faculty, staff or students of the Institute.

We'll be in the Truman Campus building of UMKC behind the Truman Library just North off Hwy 24 in Independence. We plan to have several rooms organized with physics demonstrations and laboratory equipment, mathematics problems on chalkboards, and videotapes of recent problem-solving sessions and of past enrichment speakers. In Room 102 at 2:30 pm, there will be some brief remarks by the MPI Director and the introduction of the MPI teachers, followed by a 10-minute slide-tape presentation, and of course, refreshments(!), all staffed by this year's students and faculty. We invite you all, and look forward to seeing you on:

SUNDAY, NOV. 8, 1987
2-4 PM

OUR HIGH SCHOOL TEACHERS 1987-88

From the Fort Osage District High School comes LARRY HARDING, who, along with Calvin Nelson from Northeast High School, has been one of our loyal Physics instructors all four years. (In fact, his son Brent was in our first class.) Back at 'the Fort' however he has been in the math department for 22 years, teaching the gamut of courses from Algebra to Math Analysis, and taking on both physics and advanced chemistry on occasion. He is also no stranger to high school sports, having been head basketball coach for five years, and now being golf coach.

But his interests were initially somewhat different at Kansas State University where he studied nuclear engineering for 2 1/2 years, before switching to Kansas State College at Pittsburg, eventually receiving there his MS in chemistry.

Past MPI students say of Larry:

"...gives us extra problems and shows us all of the possible ways to work (them)...along with explanations of <u>why</u> the problems work."

"...will explain answers in great detail, usually on the board."

"Gives me a great understanding."

"I couldn't have made it without him."

It is certainly in part this variety of skills that has brought flexibility and clarity to his teaching at the MPI.

INTERNATIONALLY-KNOWN MATHEMATICIAN PETER HILTON VISITS

On Thurs. Oct. 29, in a special two-hour enrichment, the MPI will be proud to once again host talks by Peter Hilton and

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Jean Pedersen, both mathematicians who have visited us once before in Fall 1985, our second year. Luckily they will be attending a conference in Kansas City in late October and have graciously found the time to address our students.

Peter is Distinguished Professor of Mathematics at the State University of New York at Binghamton, and has lectured around the world on his research as well as on mathematical education. He has written numerous papers and several books, some with his co-worker, Jean Pedersen. Jean teaches at the University of Santa Clara, Ca., is an associate editor of Mathematics Magazine, and has strong interests in polyhedral geometry as well as mathematics education.

Their topic this year will be: The Binomial Theorem and Patterns in Pascal's Triangle. (We videotaped their last visit and these tapes have become part of our video library, which is open to any Missouri school.) The next newsletter will contain a report of their visit and student reaction to it.

MORE ENRICHMENTS

Our schedule has altered some since the last newsletter:

Sept. 30 brings Dr. John Mischler, Vice-Chancellor for Research at UMKC to discuss: Careers in Research. Dr. Mischler is by training a biologist and we look forward to an enlightening visit.

Oct. 14 will be the date Buford Baber, Director of Student Financial Aid at UMKC, will address the large topic of College Financial Aid, a talk requested by students from last year.

On Thurs., Oct. 29 we'll have a special two-hour visit from Peter Hilton and Jean Pedersen. (See the article on their visit elsewhere in this issue.)

Nov. 11 we are hoping once again to visit the Research Nuclear Reactor in Columbia, Mo., which was one of the favorite trips of last year's students.

Just before Thanksgiving, on Nov. 25, one of our most popular speakers, Dr. Henry Mitchell, Associate Vice-Chancellor at UMKC, and professor of biology, will return for the fourth time to present his lecture on "Bats". And believe it or not, this time we have high hopes that the season will be right for him to bring along a live bat!

A SOLUTION TO
MATHEMATICS CHALLENGE #1

Recall the problem statement:

The sum of a certain number of consecutive positive integers is 1000. Find ALL SUCH SETS of consecutive positive integers

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ONE SOLUTION:

(a)

Suppose such a set begins with K and consists of $N + 1$ numbers, where of course: $K, N + 1 \geqslant 1$. Then:

$$1000 = K + (K + 1) + \dots + (K + N)$$

To combine the right side of this equation, first add up the K's; there are $N + 1$ of them. Then, add up the remaining $1 + \dots + N$ (using the standard sum formula) to get:

$$1000 = (N + 1)K + \frac{N(N + 1)}{2},$$

$$2000 = (N + 1)(2K + N).$$

Factor $2000 = 2^4 \cdot 5^3 = 16 \cdot 125$. So this last equation becomes:

$$16 \cdot 125 = (N + 1)(2K + N) \quad (*)$$

[PAUSE: So far we've just simplified the question in order to clearly understand it. Now it seems natural to consider cases when the factors on the right are odd or even. But first, an easy observation will shorten this work later.]

Since $K \geq 1$, we have $2K \geq 2$, so $N + 1 < N + 2K$. Since these factors from the right side of the (*)-equation above are not equal, one of them must be larger than the square root of 2000, and one must be smaller, so we can write the inequalities:

$$N + 1 \leq 44 < \sqrt{2000} < 45 \leq N + 2K \quad (\wedge)$$

(b)

Now either N is even, or N is odd:

(i) 'N is even' means, looking back to the (*)-equation, that N + 1 is odd and can only divide into 125, hence N + 1 = 1, 5, 25, or 125. But from the (^)-equation we must have: $N + 1 \leq 44$. So N + 1 cannot be 125, forcing N = 0, 4, or 24 only. Substituting these values into the (*)-equation gives three solutions:

$N = 0$: $2000 = 1(0 + 2K)$,
$1000 = K$.

$N = 4$: $2000 = 5(4 + 2K)$,
$400 = 4 + 2K$,
$198 = K$.

$N = 24$: $2000 = 25(24 + 2K)$,
$80 = 24 + 2K$,
$28 = K$.

(ii) 'N is odd' means, looking back to the (*)-equation again, that N + 2K must be odd, and can also only divide into 125, hence: N + 2K = 1, 5, 25, or 125. But from the (^)-equation, $45 \leq N + 2K$, so we only have the case N + 2K = 125 to check in the (*)-equation, yielding one more solution:

$$2000 = (N + 1) \cdot 125,$$
$$16 = N + 1,$$
$$15 = N,$$

and, solving for K gives:

$$15 + 2K = 125,$$
$$2K = 110,$$
$$K = 55.$$

(c)

Thus, there are only FOUR SETS, beginning with some K, of N + 1 consecutive positive integers which add to 1000, as follows:

1000 = 1000,
= 198 + 199 + 200 + 201 + 202,
= 28 + 29 + ... + 51 + 52,
and, = 55 + 56 + ... + 69 + 70.

DISCUSSION:

Two main facts were used here. The first is straightforward:

(i) A positive integer is either even (divisible by 2) or odd (NOT divisible by 2).

The second is only a bit more subtle:

(ii) If $ab = c$, where $a,b,c > 0$, and, say, $a \leq b$, then:

$$a \leq \sqrt{c} \leq b.$$

Proof: Suppose, by way of contradiction, that $a > \sqrt{c}$. Then,

$$a > \sqrt{c}$$
$$\sqrt{c} \cdot a > \sqrt{c} \cdot \sqrt{c} = c = ab.$$

Dividing by a (which is > 0),

$$\sqrt{c} > b \geqslant a$$

where the last inequality is by the assumption in (ii). But this contradicts our supposition that $a > \sqrt{c}$!

Likewise, supposing $\sqrt{c} > b$ leads to a similar contradiction. (Write it out!)//

Each of these facts, like so many in mathematics, is simple but surprisingly effective.

Incidentally, the second 'fact', (ii), was proven true by showing that it CANNOT be false, in other words, by showing that the OPPOSITE statements about the relations of a to $\sqrt{c}$ and $\sqrt{c}$ to b lead to contradictions. This is a VERY common type of logical argument in mathematics and bears looking at closely.

MATHEMATICS CHALLENGE #2

PROVE that if the sides of a rectangle R are positive integers a and b, then the Perimeter of R = the Area of R only when this common value is either 16 or 18.

Feel free to send solutions. A solution and discussion will be printed in the December 1 issue.

WHERE, OH WHERE ARE THE. TEACHING ASSISTANTS?

As this newsletter goes to press, the tutoring program remains on 'hold' due to funding difficulties. However, assistance IS available, and students are encouraged to speak out when they need support.

Willing helpers include peers (phone numbers have been handed out), MPI high school instructors (all are willing to make time), instructors in high schools but not at the Institute (two helping that we know of are Ken Musgrave, calculus, Fort Osage, and Ed Crawford, physics, Van Horn), and the college instructors; Richard Waring and Richard Delaware will meet students before and after classes on a drop-in basis and after high school hours by appt. Potential TA Joe Doau is currently donating his time to the MPI and is helping students after hours by phone. Students, if you are stuck, ASK!

Teddy Lambson

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The MPI Newsletter is published on the first of the month during the months of August, October, December, February, and April at The Mathematics and Physics Institute, 600 W. Mechanic, Independence, Mo. 64050, phone (816) 276-1272. Please address all correspondence concerning this newsletter to 'Newsletter'.