25 October 2007
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Images taken by Linda Hall Library; Historical notes from Bruce Bradley, and :

- Katz, Victor J., A History of Mathematics: An Introduction, 2nd Ed, 1998

Contents

Chronological by date of first editions

PAGE

Euclid, c. 300 BCE
The Elements of Geometry

First Printed Edition, in Latin, 1482

Elementa geometria. Venice: Erhard Ratdolt, 1482.

After Gutenberg's invention of the printing press, Euclid's *Elements of Geometry* was the first mathematical work to be printed, and the first major work to be illustrated with mathematical diagrams.

First English Translation, 1570

The Elements of geometrie. London: Imprinted ... by John Daye, 1570.

The 1570 publication of this book, also known as the Billingsley translation, contains a preface by John Dee who also added annotations and additional theorems. This edition is especially noted for the addition of pop-ups, to illustrate problems of solid geometry as three-dimensional figures in book eleven.

Euclid in Color, 1847

Byrne, Oliver,
The first six books of the elements of Euclid, in which coloured diagrams and symbols are used instead of letters for the greater ease of learners. London: William Pickering [of P & Chatto], 1847.

Oliver Byrne's edition of Euclid's *Elements* substituted colors for the usual letters to designate the angles and lines of geometric figures, making it one of the oddest and most beautiful mathematical books ever printed. It was also one of the most difficult to print, as the use of color wood blocks required exact registration to correctly align the pages for pass through the printing press for the different colors of ink.

Byrne's Euclid
http://www.sunsite.ubc.ca/DigitalMathArchives/Euclid/byrne.html

With live Java diagrams
http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

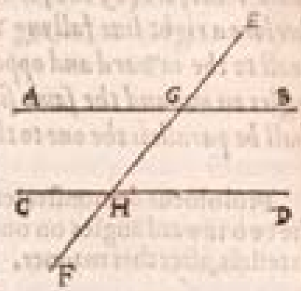
The Visual Elements of Euclid
http://www.visual-euclid.org

Euclid 1570

ble. Wherfore it is not possible that the inward angles being equal to two right angles, the right lines should concurre. Wherefore they are parallels: which was required to be proued.

The 20. Theoreme. The 29. Proposition.

A right line line falling vppon two parallel right lines: maketh the alternate angles equall the one to the other: and also the outwarde angle equall to the inwarde and opposite angle on one and the same side: and moreouer the inwarde angles on one and the same side equall to two right angles.



SVppose that vpon these parallel lines A B and C D do fal the right line E F. Then I say that the alternate angles which it maketh, namely, the angles A G H and G H D, are equall the one to the other. and ẏ the outward angle E G B is equal to the inwarde and opposite angle on the same side, namely, to ẏ angle G H D: and ẏ the inward angles on one and the selfe same side, that is, the angles B G H and G H D, are equall to two right angles. For if the angle A G H be not equal to the angle G H D, the one of them is greater. Let the angle A G H be greater. And forasmuch as the angle A G H is greater then the angle G H D, put the angle B G H commō to thē both. Wherfore ẏ angles A G H and B G H, are greater thē ẏ angles B G H & G H D. But by ẏ 13. propopositiō ẏ angles A G H & B G H are equall to two right angles, wherfore ẏ angles B G H & G H D are lesse thē two right angles. But (by ẏ 5. peticion) if vpō two right lines do fall a right line, making ẏ inward angles on one and ẏ same side, lesse thē two right angles, those right lines being infinitly produced must needes at ẏ length meete on the side wherin are the angles lesse thē two right angles. Wherfore the right lines A B and C D being infinitely produced will at ẏ length meete, But they cannot meete, because they are parallels (by supposition): wherfore the angle A G H is not vnequall to the angle G H D: wherfore it is equall.

Demonstration leading to an impossibility. First part.

And the angle A G H is (by the 15. proposition) equall to the angle E G B. Wherfore (by the first common sentence) the angle E G B is equall to the angle G H D.

Second part.

Put the angle B G H common to them both: wherfore the angles E G B and B G H are equall to the angles B G H and G H D. But the angles E G B

Third part.

and

Euclid 1570 (con.)

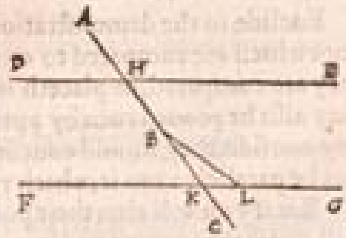
and B G H are (by the 13. proposition) equall to two right angles. VVherefore the angles B G H and G H D are also equall to two right angles. If a lyne therfore do fall vpō two parallel right lines: it maketh the alternate angles equal the one to the other: and also the outward angle equall to the inward and opposite angle on one and the same side: and moreouer the inward angles on one and the same syde equall to two right angles: which was required to be demonstrated.

This proposition is the conuerse of the two propositions next going before. For, that which in either of them is the thing sought, or cōclusion, is in this the thing geuen, or supposition. And contrariwise the thinges which in them were geuen or suppositions, are in this proued, and are conclusions. *This proposition is the conuerse of the two former propositions.*

Pelitarius after this proposition addeth this witty conclusion.

If two right lines which cut two parallel lines, do betwene the sayde parallel lines concurre in a point, and make the alternate angles equall, or the outward angle equall to the inward and opposite angle on the same side, or finally the two inward angles on one and the selfe same side equall to two right angles: those two right lines are drawen directly and make one right line. *An addition of Pelitarius.*

Suppose that there be two right lines *A B* and *C B*, which let cut two parallel lines *D E* and F G: and let *A B* cut the line *D E* in the point *H*: and let *C B* cut the line F G in the point *K*: & let the lines *A B* & *C B*, concurre betwene the two parallel lines *D E* & F G in the point *B*: and let the angle *D H B* be equal to the angle B K G: or let the angle *A H D* be equall to the angle B K F: or finally let the angles *B H D* and B K F be equal to two right angles. Thē I say that the two lines *A B* and *B C* are drawen directly, and do make one right line. For if they be not, then produce *A B* vntil it cut F G in the point L, and let A L be one right line, and so shal be made the triangle *B L K*. Now then (by the first part of this 29. proposition) the angle *D H B* shalbe equal to the alternate angle *G L B*: but (by supposition) the angle *D H B* is equall to the angle *B K G*. Wherefore the angle *B L G* is equall to the angle *B K L*, namely, the outward angle to the inwarde and opposite angle: which (by the 16. proposition) is impossible. *Demonstration leading to an absurditie. First part.*



Moreouer (by the secōd part of this 29. propositiō) the angle A H D shalbe equal to the angle *B L K*, namely, the outward angle to the inward and opposite angle on one and the same side. But the same angle *A H D* is supposed to be equall to the angle B K F: wherefore the angle B K F is equall to the angle *B L K*. Which (by the selfe same 16. proposition) is impossible. *Second part.*

Lastly forasmuch as the angles *B H D* and B K F are supposed to be equall to two right angles, & the angles *B H D* & B L K are also by the last part of this 29 proposition equal to two right angles: therefore the angle B K F shalbe equal to the angle B L K: which agayne by the selfe same 16. proposition is impossible. *Third part.*

The 21. Theoreme *The 30. Proposition.*

Right lines which are parallels to one and the selfe same right line: are also parrallel lines the one to the other.

Byrne's Euclid 1847

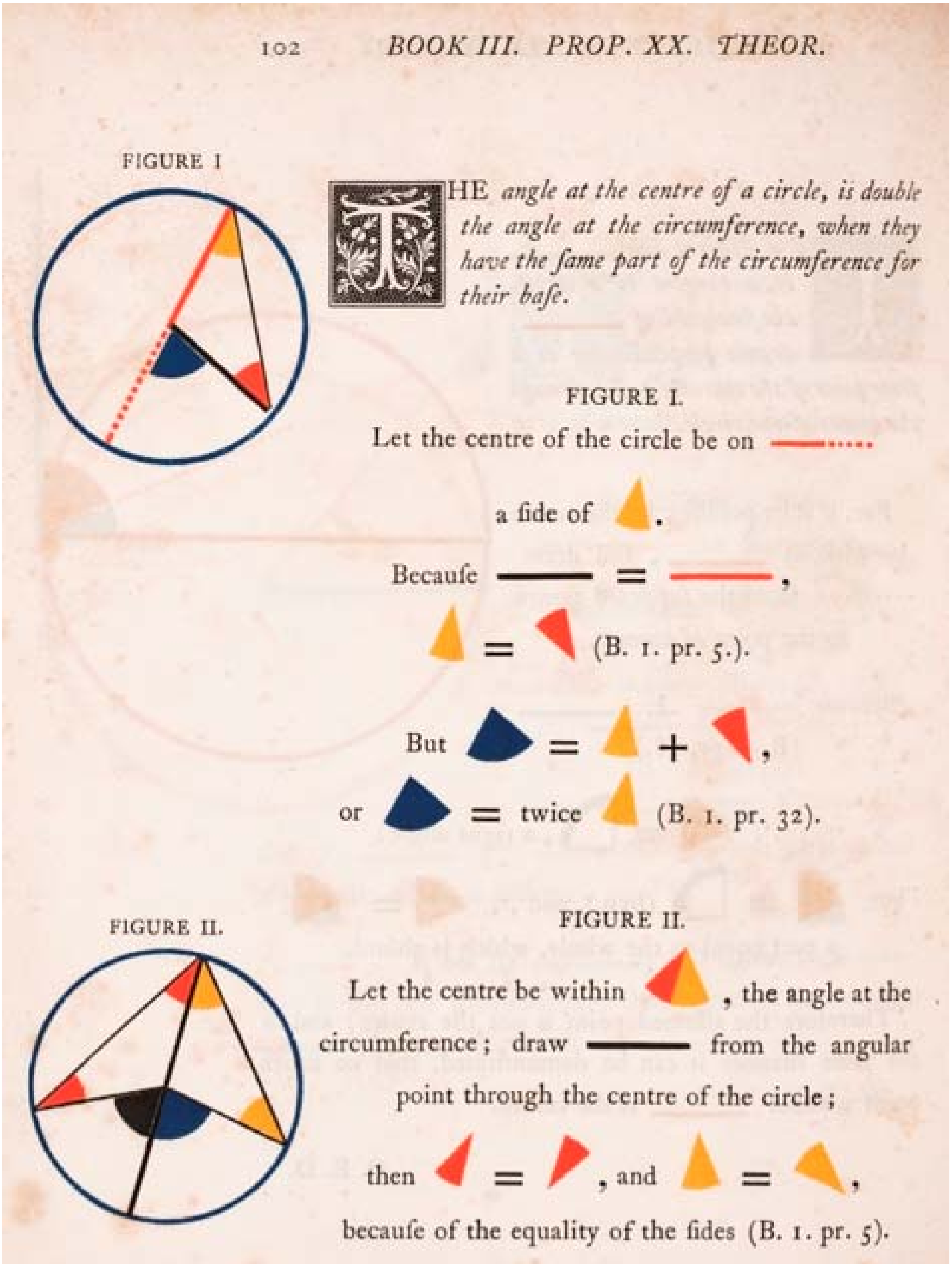


FIGURE I

THE *angle at the centre of a circle, is double the angle at the circumference, when they have the ſame part of the circumference for their baſe.*

FIGURE I.

Let the centre of the circle be on

a ſide of .

Becauſe = ,

= (B. 1. pr. 5.).

But = + ,

or = twice (B. 1. pr. 32).

FIGURE II.

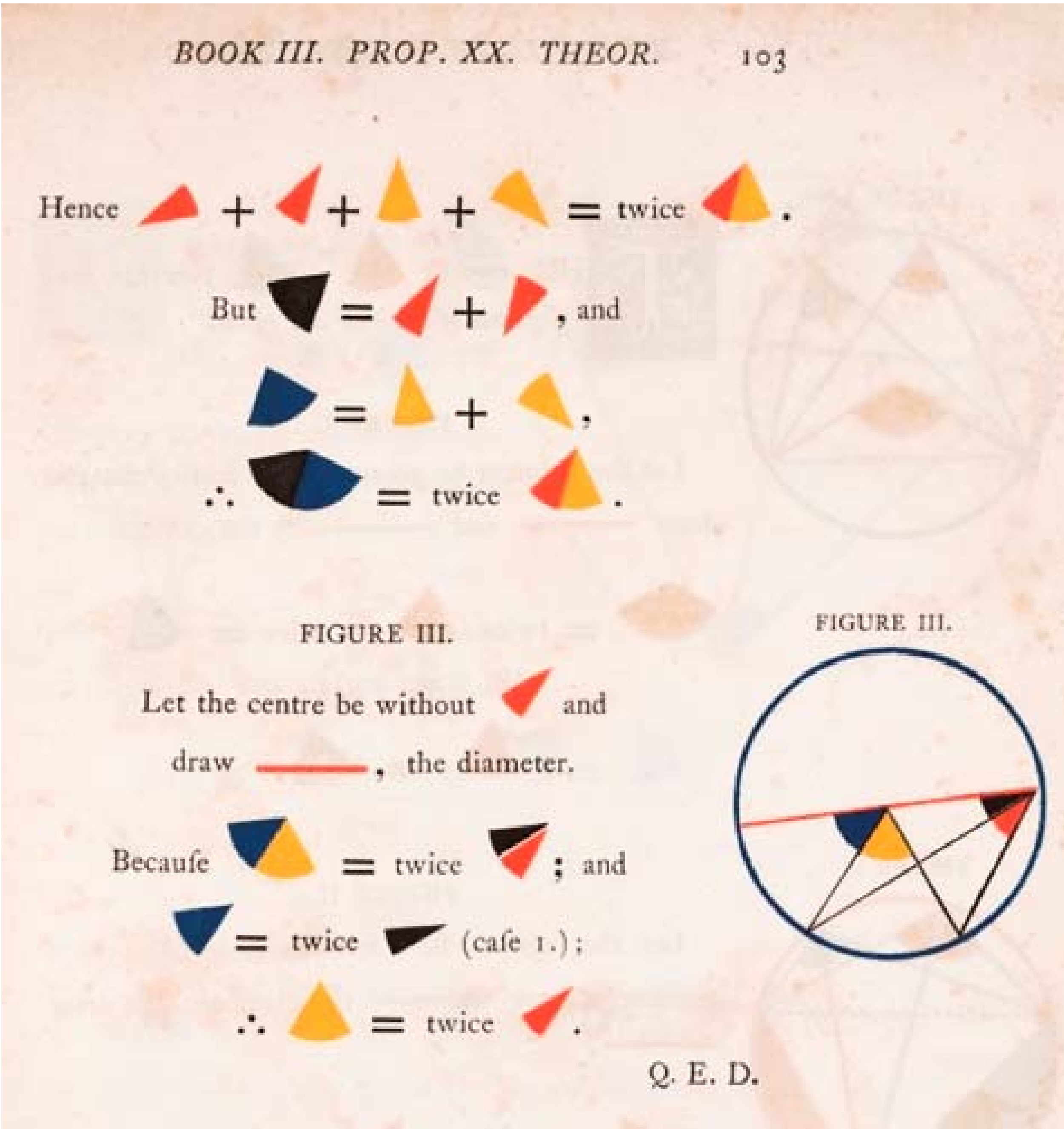
FIGURE II.

Let the centre be within , the angle at the circumference; draw from the angular point through the centre of the circle;

then = , and = ,

becauſe of the equality of the ſides (B. 1. pr. 5).

Byrne's Euclid 1847 (con.)



BOOK III. PROP. XX. THEOR. 103

Hence + + + = twice .

But = + , and

= + ,

∴ = twice .

FIGURE III.

Let the centre be without and

draw , the diameter.

Becaufe = twice ; and

= twice (cafe 1.);

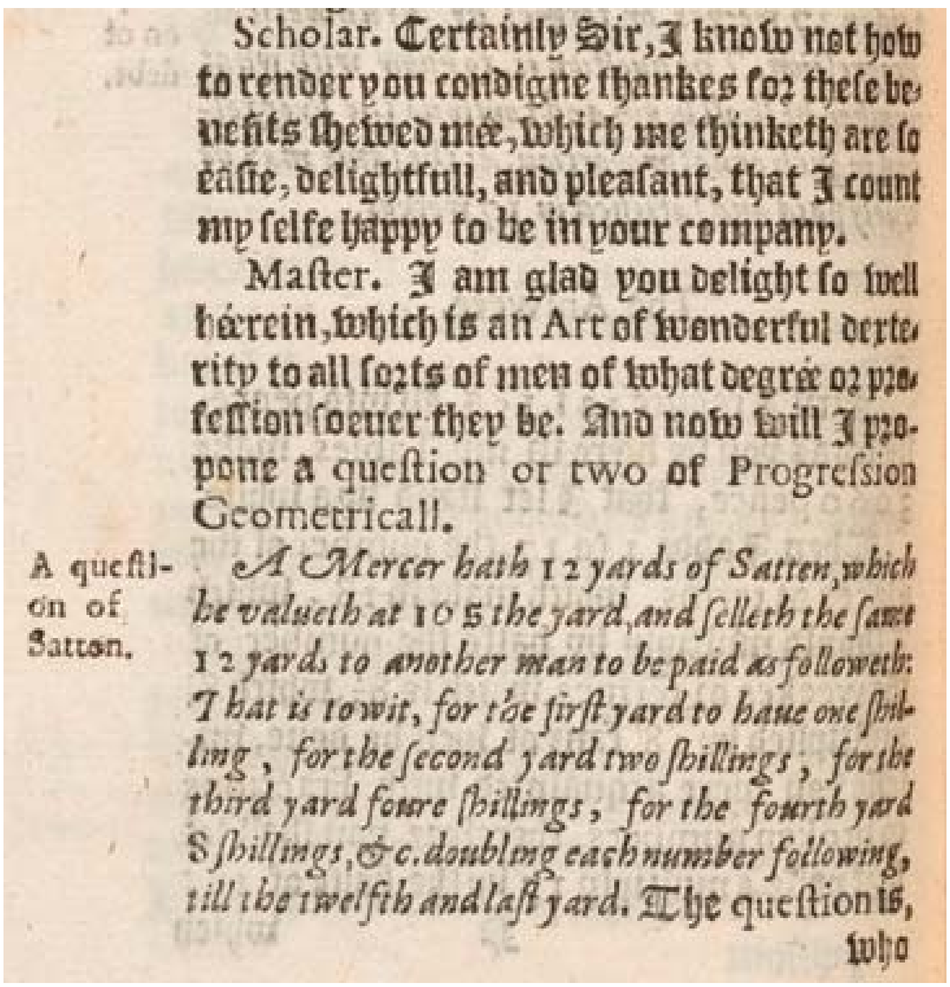
∴ = twice .

Q. E. D.

Robert Recorde, 1510-1558
The Ground of Artes
1623 (first edition 1542)

Ground of arts; teaching the perfect works and practice of arithmeticke, both in whole numbers and fractions...afterwards augmented by John.. Dee, and since enlarged ... by John Mellis...[and] Robert Hartwell.
London: John Beale for Roger Jackson, 1623.

Record wrote several books on mathematical subjects, mainly in the form of a dialog between a master and a scholar. The *Grounde of Artes* first appeared in 1542 and included only a section on Arithmetic. In 1557, he published the first book on algebra in English, The *Whetstone of Witte*, made famous by his invention of a symbol (=) to express equality. In this 1623 edition of the *Grounde of Artes* (one of at least 27 early editions!), the book on algebra was included as the second part. Ironically, the equal symbol was not used by the editors, and only slowly came into common use later in the seventeenth century.


Scholar. Certainly Sir, I know not how to render you condigne thankes for theſe benefits ſhewed mée, which me thinketh are ſo éaſie, delightfull, and pleaſant, that I count my ſelfe happy to be in your company.

Maſter. I am glad you delight ſo well héerein, which is an Art of wonderful dexterity to all ſorts of men of what degrée or profeſſion ſoeuer they be. And now will I propone a queſtion or two of Progreſsion Geometricall.

A queſti-on of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 10 s the yard, and ſelleth the ſame 12 yards to another man to be paid as followeth: That is to wit, for the firſt yard to haue one ſhilling, for the ſecond yard two ſhillings, for the third yard foure ſhillings, for the fourth yard 8 ſhillings, &c. doubling each number following, till the twelfth and laſt yard. The queſtion is, who

who hath made the better bargaine of the buyer or the ſeller.

Firſt you may ſet downe 12, the number of the yards, as you ſée héere in this Example. And againſt each number the number of ſhillings due to be paid, as the order of Duplation or Multiplication by two teacheth.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

Then reſorting to the adding vp or ſumming of this Progreſsion, where I conſider that the increaſe of this ſum procéeded by the Multiplication of 2, and therefore after I haue drawne a line vnder the 12, I worke and multiply the laſt ſumme by 2 alſo, and it yéeldeth 4096: from whence I abate the firſt number of the Progreſsion, which is 1, and then reſteth 4095: which I ſhould diuide by one leſſe then I did multiply by, but ſéeing it is 1, I néede not to diuide it: for 1 (as I haue ſaid before) doth neither multiply nor diuide; therefore I take that ſumme 4095 for the whole ſumme of the ſhillings, which by Reduction amounteth to 204 l', 15 s: and ſo much hath the Mercer for his twelue yards of Satten: which is 17 l', 1 s, 3 d, a yard.

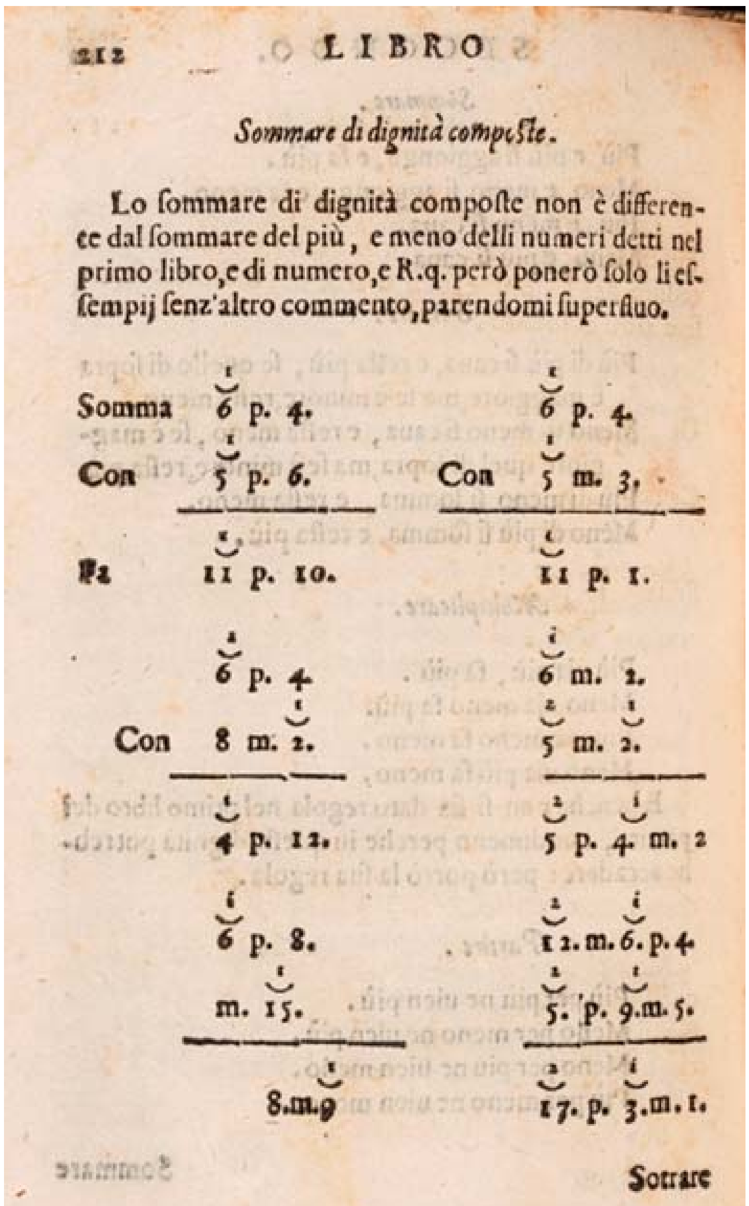
But I thinke you will buy none ſo deare.

Scholar. No ſir by the grace of God this yeare.

Raffaele Bombelli, 1526-1572
Algebra
1579 (original edition 1572)

L'algebra opera di Rafael Bombelli da Bologna. Diuisa in tre libri. Bologna: Per Giouanni Rossi, 1579

Bombelli's was not the first book on algebra. In 1545, Girolamo Cardano explored the subject in depth in his treatise, the *Ars Magna*. But Cardano's was a difficult book, Bombelli judged, and the world needed an algebra text that would allow anyone to master the subject. So Bombelli wrote one between 1557 and 1560, finally printed in 1572, intended as a systematic and logical textbook. Only three parts (of five) were published, but it was so successful that Leibniz, who used Bombelli's *Algebra* to teach himself mathematics a century later, called Bombelli the "outstanding master of the analytical art."



212 LIBRO

Sommare di dignità composte.

Lo sommare di dignità composte non è differente dal sommare del più, e meno delli numeri detti nel primo libro, e di numero, e R.q. però ponerò solo li essempij senz'altro commento, parendomi superfluo.

Somma	6^{1} p. 4.		6^{1} p. 4.
Con	5^{1} p. 6.	Con	5^{1} m. 3.
Fa	11^{1} p. 10.		11^{1} p. 1.
	6^{2} p. 4		6^{1} m. 2.
Con	8 m. 2^{1}.		5^{2} m. 2^{1}.
	4^{1} p. 12.		5^{2} p. 4^{1} m. 2
	6^{1} p. 8.		12^{2} m. 6^{1} p. 4
	m. 15^{1}.		5^{2} p. 9^{1} m. 5.
	8 m. 9^{1}		17^{2} p. 3^{1} m. 1.

Sotrare

Sotrare di dignità composte.

Lo sotrare di dignità composte non è differente da sotrare di p. e m. detto nel primo libro, e come si è proceduto nel sommare, così si farà nel sotrare le figure senz'altro comento.

Di	$4\overset{1}{\smile}$ p. 6.	$4\overset{1}{\smile}$ p. 6.
Caua	$2\overset{1}{\smile}$ p. 5.	$5\overset{1}{\smile}$ m. 8.
Resta	$2\overset{1}{\smile}$ p. 1.	m. $1\overset{1}{\smile}$ m. 2.

$$
\begin{array}{l}
5\overset{2}{\smile}\ \text{m.}\ 8\overset{1}{\smile}\ \text{p.}\ 2. \\
4\overset{2}{\smile}\ \text{p.}\ 6\overset{1}{\smile}\ \text{m.}\ 1\overset{3}{\smile} \\
\hline
1\overset{2}{\smile}\ \text{m.}\ 14\overset{1}{\smile}\ \text{p.}\ 1\overset{3}{\smile}\ \text{p.}\ 2.
\end{array}
$$

Moltiplicare di Dignità composte.

Moltiplichisi $4\overset{1}{\smile}$ via $6\overset{1}{\smile}$ p.8. farà $24\overset{2}{\smile}$ p. $32\overset{1}{\smile}$, e questo si fa semplicemente moltiplicando $4\overset{1}{\smile}$ via $6\overset{1}{\smile}$ fanno $24.\overset{2}{\smile}$, e moltiplicando 8 uia $4\overset{1}{\smile}$ fanno $32\overset{1}{\smile}$, che aggionti con $24\overset{2}{\smile}$ fanno $24\overset{2}{\smile}$.p.$32\overset{1}{\smile}$, e questo è il produtto.

Moltiplichisi $6\overset{1}{\smile}$ uia 7.m. $2\overset{1}{\smile}$ prima si moltiplica

$6\overset{1}{\smile}$

$6\overset{1}{\smile}$ uia 7 fa $42\overset{1}{\smile}$, e poi si moltiplica $6\overset{1}{\smile}$ uia m.$2\overset{1}{\smile}$, fa m.$12\overset{2}{\smile}$, che aggionti con $42\overset{1}{\smile}$ farà $42\overset{1}{\smile}$ m.$12\overset{2}{\smile}$.

Moltiplichisi $6\overset{1}{\smile}$ p.2. uia $6\overset{1}{\smile}$ p.2. Pongasi in regola (come si uede) poi si moltiplica p.2. di sotto uia p. 2. di sopra, fa p. 4, e questo si pone sotto la prima linea, poi si moltiplica p. 2. di sotto via p. $6\overset{1}{\smile}$ di sopra, fa $12\overset{1}{\smile}$, e si pone sotto la linea, poi si moltiplica $6\overset{1}{\smile}$ di sotto via 2 di sopra fa p. $12\overset{1}{\smile}$, e questo si pone sotto la linea, poi si moltiplica $6\overset{1}{\smile}$ di sotto uia $6\overset{1}{\smile}$ di sopra, fa $36\overset{2}{\smile}$, qual si pone sotto la linea, e si hauerà $36\overset{2}{\smile}$ p.$12\overset{1}{\smile}$ p.$12\overset{1}{\smile}$ p.4. E perche p.$12\overset{1}{\smile}$ ui è due uolte, si gionghino insieme, e faranno $24\overset{1}{\smile}$, si che tutta la somma (come si uede sotto la seconda linea) sarà $36\overset{2}{\smile}$ p.$24\overset{1}{\smile}$ p.4, E questo sarà il produtto della moltiplicatione.

$$\begin{array}{l}
\overset{1}{\smile}\\
6.\ \text{p.}\ 2.\\
\overset{1}{\smile}\\
6.\ \text{p.}\ 2.\\
\hline
36\overset{2}{\smile}\ \text{p.}\ 12\overset{1}{\smile}\ \text{p.}\ 12\overset{1}{\smile}\ \text{p.}\ 4.\\
\hline
36\overset{2}{\smile}\ \text{p.}\ 24\overset{1}{\smile}\ \text{p.}\ 4.
\end{array}$$

Moltiplichisi $6\overset{1}{\smile}$ p. 2 uia $6\overset{1}{\smile}$ m. 2. Pongasi in regola, poi si moltiplichi m. 2. di sotto uia p. 2. di sopra, fa m. 4, e poi si moltiplichi m. 2. di sotto via p $6\overset{1}{\smile}$ di sopra farà m. $12\overset{1}{\smile}$, poi si moltiplichi 6

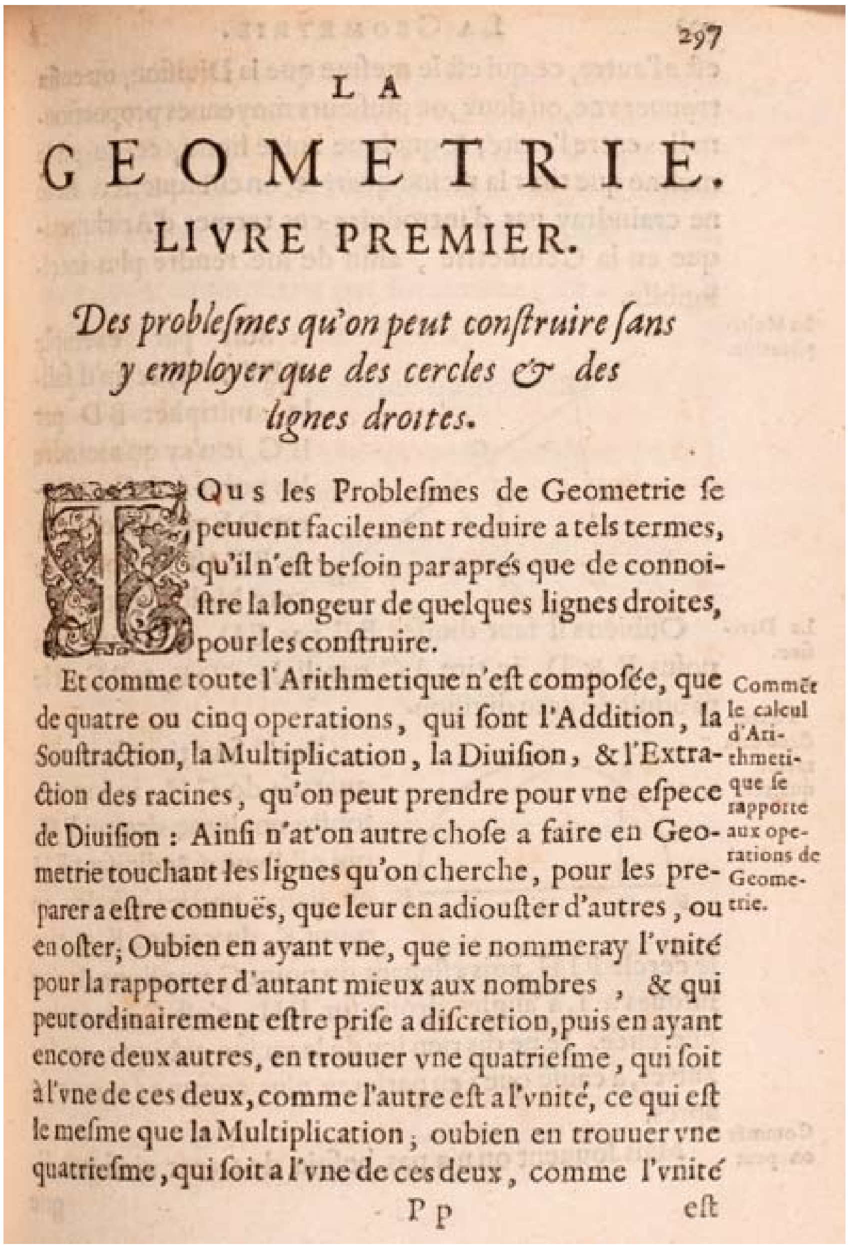
$$\begin{array}{l}
\overset{1}{\smile}\\
6.\ \text{p.}\ 2.\\
\overset{1}{\smile}\\
6.\ \text{m.}\ 2.\\
\hline
36\overset{2}{\smile}\ \text{p.}\ 12\overset{1}{\smile}\ \text{m.}\ 12\overset{1}{\smile}\ \text{m.}\ 4.\\
\hline
36\overset{2}{\smile}\ \text{m.}\ 4.
\end{array}$$

$\overset{1}{\smile}$ di sotto uia p.2. di sopra, fa p.$12\overset{1}{\smile}$ e poi $6\overset{1}{\smile}$ di sotto via $6\overset{1}{\smile}$ di sopra, fa $36\overset{2}{\smile}$, e tutte queste moltiplicationi poste sotto la linea saranno $36\overset{2}{\smile}$ p.$12\overset{1}{\smile}$ m. $12\overset{1}{\smile}$ m.4. E per esserci p.$12\overset{1}{\smile}$ e m.$12\overset{1}{\smile}$ si leuano per le regole date del p.& m. e restaranno $36\overset{2}{\smile}$ m. 4 (come si vede) per produtto della moltiplicatione.

René Descartes, 1596-1650
Geometry, (an Appendix to Discourse on Method)
1637

Discours de la methode pour bien conduire sa raison, & chercher la verite dans les sciences ... Plus la Dioptrique. Les meteores. Et la geometrie. Leiden: De l'imprimerie de Ian Maire, 1637.

Descartes' *Geometry* first appeared in French as an appendix to a larger work called *Discourse on the Method of Properly Conducting One's Reason and of Seeking the Truth in the Sciences.* The appendix on geometry was meant to illustrate the effectiveness of the method laid out in the *Discourse*. “Any problem in geometry,” Descartes began, “can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction.”



297

LA

GEOMETRIE.

LIVRE PREMIER.

Des problesmes qu'on peut construire sans y employer que des cercles & des lignes droites.

TOus les Problesmes de Geometrie se peuuent facilement reduire a tels termes, qu'il n'est besoin par aprés que de connoistre la longeur de quelques lignes droites, pour les construire.

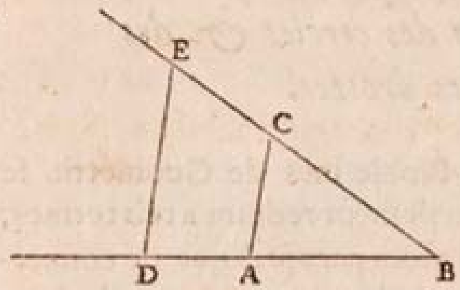
Et comme toute l'Arithmetique n'est composée, que de quatre ou cinq operations, qui sont l'Addition, la Soustraction, la Multiplication, la Diuision, & l'Extraction des racines, qu'on peut prendre pour vne espece de Diuision : Ainsi n'a-t'on autre chose a faire en Geometrie touchant les lignes qu'on cherche, pour les preparer a estre connuës, que leur en adiouster d'autres, ou en oster; Oubien en ayant vne, que ie nommeray l'vnité pour la rapporter d'autant mieux aux nombres , & qui peut ordinairement estre prise a discretion, puis en ayant encore deux autres, en trouuer vne quatriesme, qui soit à l'vne de ces deux, comme l'autre est a l'vnité, ce qui est le mesme que la Multiplication; oubien en trouuer vne quatriesme, qui soit a l'vne de ces deux, comme l'vnité

Commẽt le calcul d'Arithmetique se rapporte aux operations de Geometrie.

Pp est

est a l'autre, ce qui est le mesme que la Diuision; ou enfin trouuer vne, ou deux, ou plusieurs moyennes proportionnelles entre l'vnité, & quelque autre ligne; ce qui est le mesme que tirer la racine quarrée, ou cubique, &c. Et ie ne craindray pas d'introduire ces termes d'Arithmetique en la Geometrie, affin de me rendre plus intelligibile.

La Multiplication.

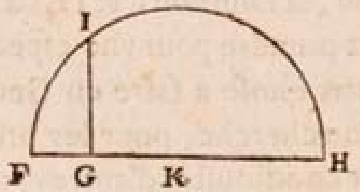


Soit par exemple A B l'vnité, & qu'il faille multiplier B D par B C, ie n'ay qu'a ioindre les poins A & C, puis tirer D E parallele a C A, & B E est le produit de cete Multiplication.

La Diuision.

Oubien s'il faut diuiser B E par B D, ayant ioint les poins E & D, ie tire A C parallele a D E, & B C est le produit de cete diuision.

l'Extraction de la racine quarrée.



Ou s'il faut tirer la racine quarrée de G H, ie luy adiouste en ligne droite F G, qui est l'vnité, & diuisant F H en deux parties esgales au point K, du centre K ie tire le cercle F I H, puis esleuant du point G vne ligne droite iusques à I, à angles droits sur F H, c'est G I la racine cherchée. Ie ne dis rien icy de la racine cubique, ny des autres, à cause que i'en parleray plus commodement cy aprés.

Commēt on peut

Mais souuent on n'a pas besoin de tracer ainsi ces ligne gnes sur le papier, & il suffist de les designer par quelques lettres, chascune par vne seule. Comme pour adiouster la ligne B D a G H, ie nomme l'vne a & l'autre b, & escris $a + b$; Et $a -- b$, pour soustraire b d'a; Et ab, pour les multiplier l'vne par l'autre; Et $\frac{a}{b}$, pour diuiser a par b; Et aa, ou a^2, pour multiplier a par soy mesme; Et a^3, pour le multiplier encore vne fois par a, & ainsi a l'infini; Et $\sqrt{a^2 + b^2}$, pour tirer la racine quarrée d'$a^2 + b^2$; Et $\sqrt{C.\,a^3 -- b^3 + abb}$, pour tirer la racine cubique d'$a^3 -- b^3 + abb$, & ainsi des autres.

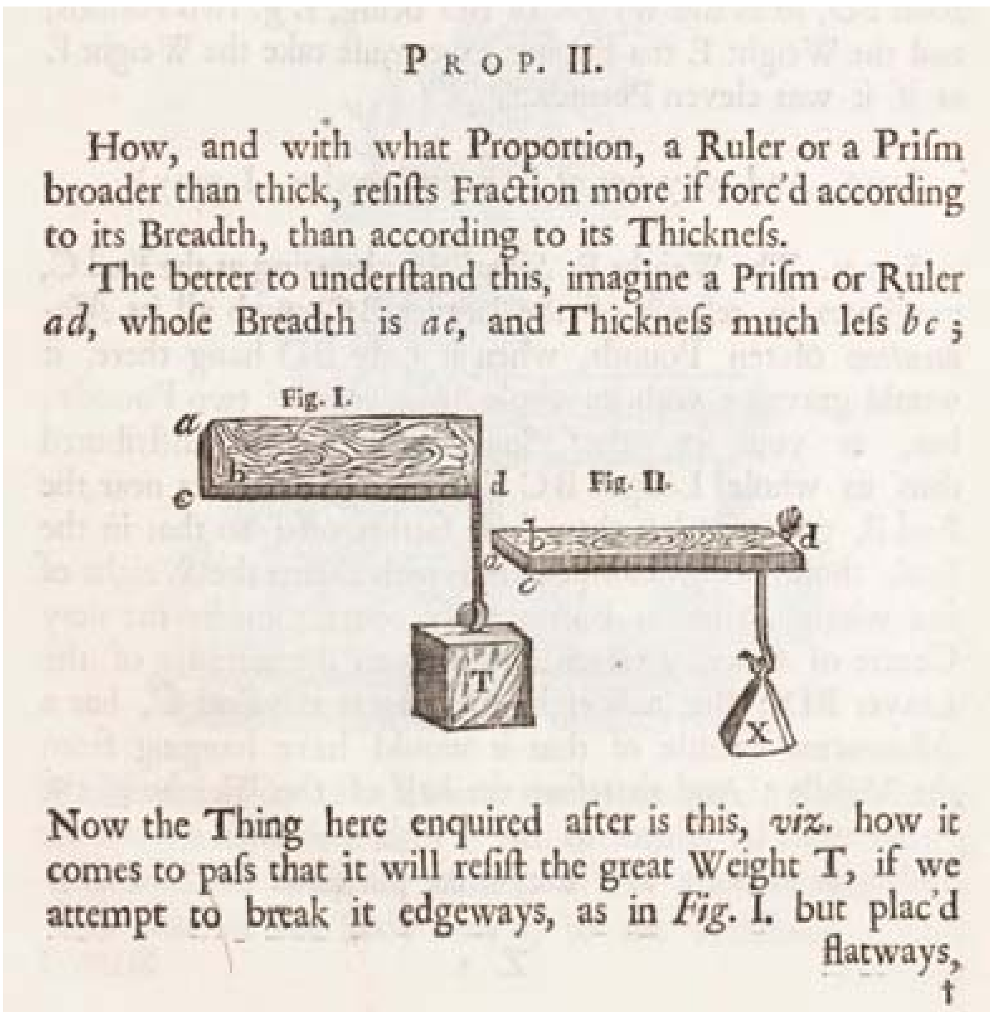
Vser de chiffres en Geometrie.

Où il est a remarquer que par a^2 ou b^3 ou semblables, ie ne conçoy ordinairement que des lignes toutes simples, encore que pour me seruir des noms vsités en l'Algebre, ie les nomme des quarrés ou des cubes, &c.

Galileo Galilei, 1564-1642
Discourses and Mathematical Demonstrations Concerning Two New Sciences
1730 (Original edition 1638)

Mathematical discourses concerning two new sciences relating to mechanicks and local motion. London: Printed for J. Hooke, 1730.

Galileo is known for his telescopic discoveries and his controversial defense of Copernicus. But when the church forbade him from further speculation and writing about astronomy, he retreated to consider problems of mathematics and engineering. The result was this book setting forth the mathematical principles of statics (the strength of materials), and kinematics (the science of bodies in motion).



PROP. II.

How, and with what Proportion, a Ruler or a Priſm broader than thick, reſiſts Fraction more if forc'd according to its Breadth, than according to its Thickneſs.

The better to underſtand this, imagine a Priſm or Ruler *ad*, whoſe Breadth is *ac*, and Thickneſs much leſs *bc*;

Now the Thing here enquired after is this, *viz.* how it comes to paſs that it will reſiſt the great Weight T, if we attempt to break it edgeways, as in *Fig.* I. but plac'd flatways,

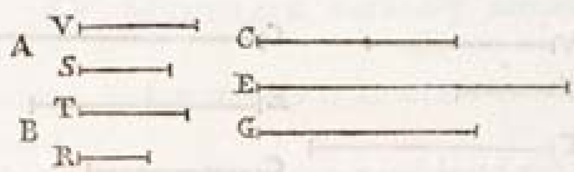
flatways, as in *Fig.* II. it will not refift the Weight X, which is lefs than T: And the Thing is plain, fince we fuppofe the *Fulcrum* in one cafe to be under the Line *ac*, and in the other, under *bc*, and the Diftances of the Forces to be equal in both Cafes, *viz.* *cd*; But in the former Cafe the Diftance of the Refiftance from the *Fulcrum*, which is half the Line *ca*, is greater than the Diftance in the other Cafe, which is half the Line *cb*: Wherefore the Power of the Weight T muft be neceffarily greater than the Weight X, as much as the half of the Breadth *ca* is greater than half the Thicknefs *bc*, that ferving for a Counter-Leaver to *ca*, and this to *cb*, to overcome the fame Refiftance, *i. e.* the Quantity of Fibres of the whole Bafe *ab*.

Wherefore we conclude that the fame Ruler or Prifm, which is broader than it is thick, refifts breaking more if plac'd edgeways than flatways; and that in Proportion of the Breadth to the Thicknefs.

PROP. V. THEOR. V.

If two Moveables move with an equable Motion, but with unequal Velocities, and if the Spaces paſſed be alſo unequal, the Proportion of the Times will be compounded of the Proportion of the Spaces, and of the Proportion of the Velocities taken reciprocally.

Let A and B be two Moveables, and let the Velocity of A be to the Velocity of B, as V to T, and let the Spaces paſſed be as S to R : Then, I ſay, the Proportion of the Time in which A is moved, to the Time in which B is moved, is compounded of the *Ratio* of the Velocity T, to the Velocity V, and of the *Ratio* of the Space S, to the Space R. Let C be the Time of the Motion A ; and as the Velocity T is to the Velocity V, ſo let the Time C be to the Time E : And ſince C is the Time wherein A, with the Velocity V, paſſes the Space S, and ſince it is as the Velocity T, of the Moveable B, to the Velocity V, ſo the Time C to the Time E, E will be the Time wherein the Moveable B would paſs thro' the ſame Space S.



Again, as the Space S is to the Space R, ſo let the Time E be to the Time G : then 'tis manifeſt, that G is the Time wherein B would paſs thro' the Space R : And becauſe the Proportion of C to G is compounded of the *Ratios* of C to E, and of E to G ; and ſince the Proportion of C to E, is the ſame with that of the Velocities of the Moveables A and B reciprocally taken, that is, with that of T and V : and ſince the Proportion of E G is the ſame with the Proportion of the Spaces S and R ; therefore the Propoſition is manifeſt.

Isaac Newton, 1642-1727
Treatise on the Method of Series and Fluxions
1736 postumously
(first edition 1671 in Latin, trans. by John Colson)
&
Mathematical Principles of Natural Philosophy
third edition, Latin
1726 (first edition 1687)

The method of fluxions and infinite series : with its application to the geometry of curve-lines. London: Printed by Henry Woodfall; and sold by John Nourse, 1736.

Newton's earliest work on the calculus, as documented in his unpublished manuscripts, came in 1665 – the same year that he took his B.A. degree. His most complete exposition on the calculus was written in 1671, in Latin, but it remained unpublished until this English translation by John Colson appeared in 1736. According to Newton, a variable was regarded as a "fluent," and thought of as a function of time, while its rate of change with respect to time was called a "fluxion." The basic problem this "calculus" was to investigate relations among fluents and their fluxions.

Philosophiae naturalis principia mathematica. London: Apud Guil. & Joh. Innys, 1726.

Neither Newton or the Royal Society had enough funds to publish the first edition of the *Principia* in 1687, the cost of which was borne by Newton's friend Edmond Halley. This third Latin edition, the last published during Newton's lifetime, became the basis for all subsequent editions. Newton was able to pay Henry Pemberton 200 guineas for his editorial assistance in seeing the work through the press.

Newton's Terms	**Newton's Notation**	**Our Notation**	**Our Terms**
Fluent	x	$x(t)$	Function of time t
Fluxion	$\dot{x}$	$\frac{dx}{dt}$	Derivative with respect to t

THE

METHOD of FLUXIONS

AND

INFINITE SERIES;

WITH ITS

Application to the Geometry of CURVE-LINES.

By the INVENTOR

Sir ISAAC NEWTON, K^t.

Late Prefident of the Royal Society.

Tranflated from the AUTHOR's LATIN ORIGINAL
not yet made publick.

To which is fubjoin'd,

A PERPETUAL COMMENT upon the whole Work,

Confifting of

ANNOTATIONS, ILLUSTRATIONS, and SUPPLEMENTS,

In order to make this Treatife

A compleat Inftitution for the ufe of LEARNERS.

By *JOHN COLSON*, M.A. and F.R.S.
Mafter of Sir *Jofeph Williamfon*'s free Mathematical-School at *Rochefter*.

LONDON:
Printed by HENRY WOODFALL;
And Sold by JOHN NOURSE, at the *Lamb* without *Temple-Bar*.
M.DCC.XXXVI.

Examples of Reduction by Division.

4. The Fraction $\frac{aa}{b+x}$ being proposed, divide aa by $b+x$ in the following manner:

$$b+x)\,aa+0\left(\frac{aa}{b}-\frac{aax}{b^2}+\frac{aax^2}{b^3}-\frac{aax^3}{b^4}+\frac{aax^4}{b^5},\ \&c.\right.$$

$$\begin{array}{l} aa+\frac{aax}{b} \\ \hline 0-\frac{aax}{b}+0 \\ -\frac{aax}{b}-\frac{aax^2}{b^2} \\ \hline 0+\frac{a^2x^2}{b^2}+0 \\ +\frac{a^2x^2}{b^2}+\frac{a^2x^3}{b^3} \\ \hline 0-\frac{a^2x^3}{b^3}+0 \\ -\frac{a^2x^3}{b^3}-\frac{a^2x^4}{b^4} \\ \hline 0+\frac{a^2x^4}{b^4}\ \&c. \end{array}$$

The Quotient therefore is $\frac{aa}{b}-\frac{a^2x}{b^2}+\frac{a^2x^2}{b^3}-\frac{a^2x^3}{b^4}+\frac{a^2x^4}{b^5}$, &c. which Series, being infinitely continued, will be equivalent to $\frac{aa}{b+x}$. Or making x the first Term of the Divisor, in this manner, $x+b)\,aa+0$ (the Quotient will be $\frac{aa}{x}-\frac{aab}{x^2}+\frac{aab^2}{x^3}-\frac{a^2b^3}{x^4}$, &c. found as by the foregoing Process.

5. In like manner the Fraction $\frac{1}{1+xx}$ will be reduced to $1-x^2+x^4-x^6+x^8$, &c. or to $x^{-2}-x^{-4}+x^{-6}-x^{-8}$, &c.

6. And the Fraction $\frac{2x^{\frac{1}{2}}-x^{\frac{3}{2}}}{1+x^{\frac{1}{2}}-3x}$ will be reduced to $2x^{\frac{1}{2}}-2x+7x^{\frac{3}{2}}-13x^2+34x^{\frac{5}{2}}$, &c.

7. Here it will be proper to observe, that I make use of x^{-1}, x^{-2}, x^{-3}, x^{-4}, &c. for $\frac{1}{x}$, $\frac{1}{x^2}$, $\frac{1}{x^3}$, $\frac{1}{x^4}$, &c. of $x^{\frac{1}{2}}$, $x^{\frac{3}{2}}$, $x^{\frac{5}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{2}{3}}$, &c. for $\sqrt{x}$, $\sqrt{x^3}$, $\sqrt{x^5}$, $\sqrt[3]{x}$, $\sqrt[3]{x^2}$, &c. and of $x^{-\frac{1}{2}}$, $x^{-\frac{2}{3}}$, $x^{-\frac{1}{4}}$, &c. for $\frac{1}{\sqrt{x}}$, $\frac{1}{\sqrt[3]{x^2}}$, $\frac{1}{\sqrt[4]{x}}$, &c. And this by the Rule of Analogy, as may be apprehended from such Geometrical Progressions as these; x^3, $x^{\frac{5}{2}}$, x^2, $x^{\frac{3}{2}}$, x, $x^{\frac{1}{2}}$, x^0 (or 1,) $x^{-\frac{1}{2}}$, x^{-1}, $x^{-\frac{3}{2}}$, x^{-2}, &c.

It may not be amiſs to give one general Example of this Reduction, which will comprehend all particular Caſes. If the Series $az + bz^2 + cz^3 + dz^4$, &c. be given, of which we are to find any Power, or to extract any Root; let the Index of this Power or Root be m. Then prepare the moveable or left-hand Paper as you ſee below, where the Terms of the given Series are ſet over one another in order, at the edge of the Paper, and at equal diſtances. Alſo after every Term is put a full point, as a Mark of Multiplication, and after every one, (except the firſt or loweſt) are put the ſeveral Multiples of the Index, as m, $2m$, $3m$, $4m$, &c. with the negative Sign — after them. Likewiſe a *vinculum* may be underſtood to be placed over them, to connect them with the other parts of the numeral Coefficients, which are on the other Paper, and which make them compleat. Alſo the firſt Term of the given Series is ſeparated from the reſt by a line, to denote its being a Diviſor, or the Denominator of a Fraction. And thus is the moveable Paper prepared.

To prepare the fixt or right-hand Paper, write down the natural Numbers 0, 1, 2, 3, 4, &c. under one another, at the ſame equal diſtances as the Terms in the other Paper, with a Point after them as a Mark of Multiplication; and over-againſt the firſt Term 0 write

write $a^m z^m$ for the firſt Term of the Series required. The reſt of the Terms are to be wrote down orderly under this, as they ſhall be found, which will be in this manner. To the firſt Term o in the fixt Paper apply the ſecond Term of the moveable Paper, and they will then exhibit this Fraction $\frac{bz^2.\ \overline{m-0}.\ a^m z^m}{az.\ 1}$, which being reduced to this $ma^{m-1}bz^{m+1}$, muſt be ſet down in its place, for the ſecond Term of the Series required. Move the moveable Paper a ſtep lower, and you will have this Fraction exhibited

$$\frac{+cz^3.\ \overline{2m-0}.\ a^m z^m \quad +bz^2.\ \overline{m-1}.\ \overline{ma^{m-1}bz^{m+1}}}{az.\ 2}$$

which being reduced will become $ma^{m-1}c + m \times \frac{m-1}{2} a^{m-2}b^2 \times z^{m+2}$, to be put down for the third Term of the Series required. Bring down the moveable Paper a ſtep lower, and you will have the Fraction

$$\frac{+dz^4.\ \overline{3m-0}.\ a^m z^m \quad +cz^3.\ \overline{2m-1}.\ \overline{ma^{m-1}bz^{m+1}} \quad +bz^2.\ \overline{m-2}.\ \overline{ma^{m-1}c + m \times \frac{m-1}{2} a^{m-2}b^2 \times z^{m+2}}}{az.\ 3}$$

which reduc'd will be $ma^{m-1}d + m \times \frac{m-1}{1} a^{m-2}bc + m \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3}b^3 \times z^{m+3}$, for the fourth Term of the Series required. And in the ſame manner are all the reſt of the Terms to be found.

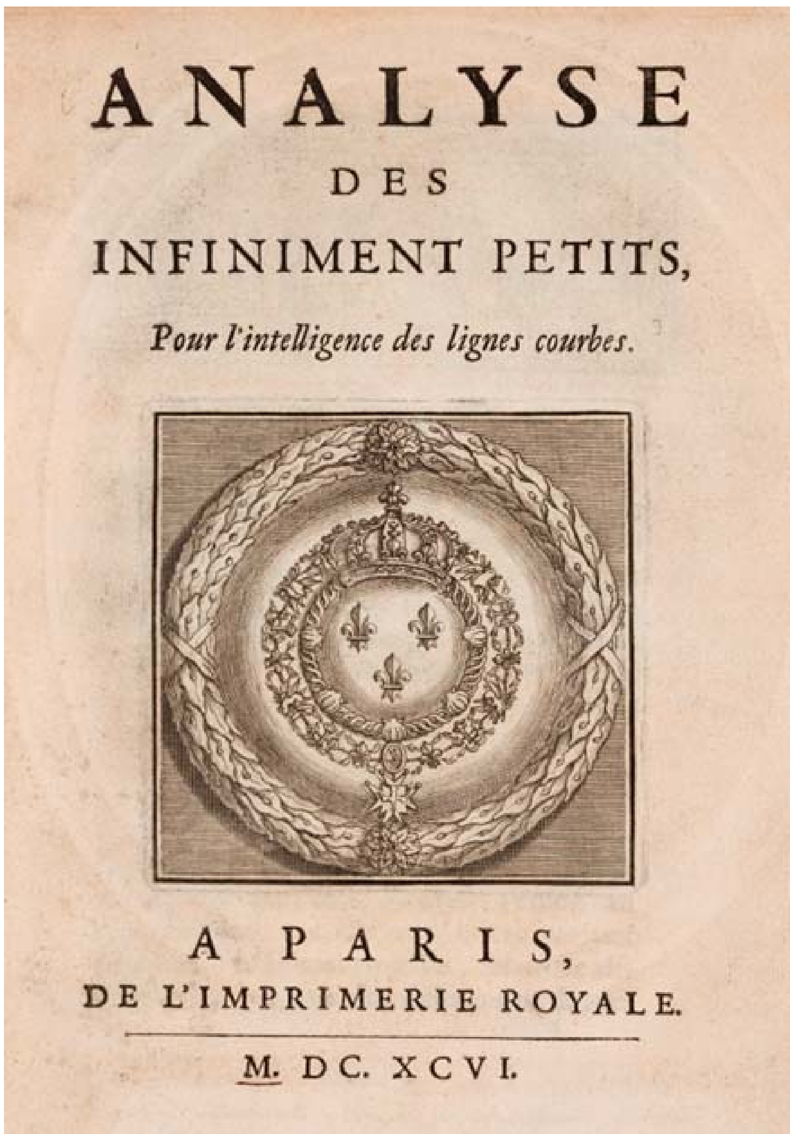
Moveable Paper, &c.
$+dz^4.\ \overline{3m-}$
$+cz^3.\ \overline{2m-}$
$+bz^2.\ \overline{m-}$
$az.$

Fixt Paper
o. $a^m z^m$
1. $ma^{m-1}bz^{m+1}$
2. $m \times \frac{m-1}{2} a^{m-2}b^2 + ma^{m-1}c \times z^{m+2}$
3. $m \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3}b^3 + m \times \frac{m-1}{1} a^{m-2}bc + ma^{m-1}d \times z^{m+3}$
&c. &c.

Guillaume François, Marquis de L'Hospital, 1661-1704
Analysis of infinitely small quantities for the understanding of curves
1696

Analyse des infiniment petits pour l'intelligence des lignes courbes. Paris: De l'Imprimerie Royale, 1696.

L'Hospital learned the new calculus from Johann Bernoulli, who spent some months in Paris teaching it to L'Hospital in 1691. Since there was no textbook on the calculus, L'Hospital wrote one. Although the *Analyse* was the first calculus textbook ever written, it has never been translated into English. L'Hospital's Rule, which he learned from Bernoulli, first appeared here.



Abraham de Moivre, 1667-1754
The Doctrine of Chances; or, a Method of Calculating the Probabilities of Events in Play
1738 (first edition 1718)

The doctrine of chances; or, a method of calculating the probabilities of events in play. -- The second edition. London: Printed for the author, by H. Woodfall, 1738.

This book on probability theory that was first published in 1718. But in the second edition of 1738 ("*Fuller, Clearer, and more Correct than the First*"), de Moivre introduced the concept of normal distributions. This is now often referred to as the theorem of de Moivre-Laplace, giving a mathematical formulation for the way that "chances" and stable frequencies are related.

PROBLEM IV.

To find how many Trials are neceſſary to make it probable that an Event will happen twice, ſuppoſing that a *is the number of Chances for its happening in any one Trial, and* b *the number of Chances for its failing.*

SOLUTION.

Let x be the number of Trials: then from what has been demonſtrated *in the* 16th *Art. of the Introd.* it follows that $b^x + xab^{x-1}$ is the

40 *The* DOCTRINE *of* CHANCES.

the number of Chances whereby the Event may fail, $\overline{a+b}|^x$ comprehending the whole number of Chances whereby it may either happen or fail, and conſequently the probability of its failing is $\frac{b^x + xab^{x-1}}{\overline{a+b}|^x}$, but by Hypotheſis the Probabilities of happening and failing are equal, we have therefore the Equation $\frac{b^x + xab^{x-1}}{\overline{a+b}|^x} = \frac{1}{2}$, or $\overline{a+b}|^x = 2b^x + 2ab^{x-1}$, or making $a, b :: 1, q$; then $\overline{1 + \frac{1}{q}}|^x = 2 + \frac{2x}{q}$. Now if in this Equation we ſuppoſe $q = 1$, x will be found $= 3$, and if we ſuppoſe q infinite, and alſo $\frac{x}{q} = z$, we ſhall have the Equation $z = \log. 2 + \log. \overline{1+z}$, in which taking the value of z, either by Trial or otherwiſe, it will be found $= 1.678$ nearly; and therefore the value of x will always be the limits 39 and 1.6789, but x will ſoon converge to the laſt of theſe limits; for which reaſon if q be not very ſmall, x may in all caſes be ſuppoſed $= 1.6789$; yet if there be any ſuſpicion that the value of x thus taken is too little, ſubſtitute this value in the original Equation $\overline{1 + \frac{1}{q}}|^x = 2 + \frac{2x}{q}$, and note the Error. Then if it be worth taking notice of, increaſe a little the value of x, and ſubſtitute again this new value of x in the aforeſaid Equation; and noting the new Error, the value of x may be ſufficiently corrected by applying the Rule which the Arithmeticians call double falſe Poſition.

A Transcription and Explication using Modern English and Notation

From: *The Doctrine of Chances*, 2nd edition, 1738, Abraham de Moivre, pp. 32-44.

Notes: All comments in smaller font and [square brackets] are mine.

PROBLEM IV.

To find how many Trials are necessary to make it equally probable that an Event will happen [at least] **twice**, supposing that *a* is the number of Chances for its happening in any one Trial, and *b* the number of Chances for its failing.

Solution.

Let *x* be the number of Trials. Then from what has been demonstrated in the 16th Article of the Introduction it follows that $b^x + xab^{x-1}$ is the number of Chances whereby the Event may fail,

[For the Event to fail to "happen [at least] **twice**", the Event must happen `at most once', meaning either exactly zero times, or exactly once. Think of *b* as the probability of failure in one trial. Since `Trials' are (by definition) independent, to calculate the probability of failure in every one of the *x* trials we multiply the individual trial probabilities, giving a total of b^x. This corresponds to the Event happening exactly zero times. If failure occurs in all but one trial, say the first trial, the probability is similarly calculated as a product to be ab^{x-1}. Since the single success (which has probability *a*) can occur in any one of the *x* trials (meaning there are *x* different placements of the success), then this probability must be added *x* times, giving xab^{x-1}. This corresponds to the Event happening exactly once, and failing the other *x*-1 times. Thus, the probability that the Event fails to "happen [at least] twice" is the sum: $b^x + xab^{x-1}$.]

$(a+b)^x$ comprehending the whole number of Chances whereby it may either happen or fail,

[Note that $(a+b)^x = \sum_{r=0}^{x} \binom{x}{r} a^{x-r} b^r = a^x b^0 + xa^{x-1}b + \cdots + xab^{x-1} + a^0 b^x$. Each term counts the number of all possible arrangements for each different assignment of failures and successes to the *x* trials. So $(a+b)^x$ turns out to be the sum of the "whole number of Chances whereby it may either happen or fail..."]

and consequently the probability of its failing is

$$\frac{b^x + xab^{x-1}}{(a+b)^x}.$$

[His definition of probability is (the number of Chances of failure) divided by (the total possible number of Chances of success or failure).]

But, by Hypothesis, the Probabilities of happening and failing are equal.
[Meaning, both must equal 1/2 (that is, they are "equally probable".)]

We have therefore the Equation

$$\frac{b^x + xab^{x-1}}{(a+b)^x} = \frac{1}{2}, \text{ or}$$

$$(a+b)^x = 2b^x + 2xab^{x-1}, \text{ or}$$

$$\text{making } \frac{a}{b} = \frac{1}{q},$$

$$\left(1+\frac{1}{q}\right)^x = 2 + \frac{2x}{q}.$$

[That is, divide the equation $(a+b)^x = 2b^x + 2xab^{x-1}$ by b^x and then substitute $\frac{a}{b} = \frac{1}{q}$.]

Now if in this Equation we suppose $q = 1$, x will be found = 3,

[Substituting $q = 1$ we get $2^x = 2 + 2x$, hence $2^{x-1} = 1 + x$.
A little guessing shows that $x = 3$ (trials) satisfies the equation. (There is no algebraic way to solve this equation, and a simultaneous graph of the functions on either side of the equality only reveals that there is a single intersection point at which x is positive.) Note that since both $a > 0$ and $b > 0$, then it follows from $\frac{a}{b} = \frac{1}{q}$ that $q > 0$. Then $q = 1$ is the smallest possible integral value of q here (its lower bound value).]

and if we suppose q infinite, and also $\frac{x}{q} = z$, we shall have the Equation

$$z = \log(2) + \log(1+z),$$

[To make sense of these statements, first observe that we can rewrite the equation above, with $\frac{x}{q} = z$, as

$$\left(1+\frac{1}{q}\right)^x = 2 + \frac{2x}{q}$$

$$\left(\left(1+\frac{1}{q}\right)^q\right)^{\frac{x}{q}} = 2\left(1+\frac{x}{q}\right)$$

$$\left(\left(1+\frac{1}{q}\right)^q\right)^z = 2(1+z).$$

Second, we recall the well-known calculus fact that as $q \to \infty$ we have

$$\left(1+\frac{1}{q}\right)^q \to e$$

where $e \approx 2.71828$ is the usual base for the natural exponential function, what he elsewhere calls the `hyperbolic base'. Thus, "if we suppose $q$ infinite" we have $e^z = 2(1+z)$ and then applying the logarithm function (to the `hyperbolic base') to both sides we find

$$\log(e^z) = \log[2(1+z)]$$
$$z = \log(2) + \log(1+z)\ .]$$

in which taking the value of z, either by Trial or otherwise, it will be found = 1.678 nearly.

[Again, there is no algebraic way to solve this equation, or the equivalent equation $e^z = 2(1+z)$. A simultaneous graph of the functions on either side of the equality once again only reveals that there is a single intersection point at which z is positive. But from this, or numerical trial and error, we can approximate the solution as $z \approx 1.678$.]

And therefore the value of x will always be between the limits $3q$ and $1.678q$, but will soon converge to the last of these limits. For which reason, if q be not very small, x may in all cases be supposed = $1.678q$.

[The equation $\frac{x}{q} = z$ can be written as $x = zq$. So from above, when $q = 1$ we found that $x = 3$, which here we write as $x = 3q$. From the last calculation, for large values of q (that is, as $q\to\infty$) we found that $x \approx 1.678q$, meaning x "will soon converge to" this value.]

Yet if there be any suspicion that the value of x thus taken is too little, substitute this value in the original Equation

$$\left(1+\frac{1}{q}\right)^x = 2+\frac{2x}{q},$$

and note the Error. Then if it be worth taking notice of, increase a little the value of x, and substitute again this new value of x in the aforesaid Equation. And noting the new Error, the value of x may be sufficiently corrected by applying the Rule which the Arithmeticians call double false Position.

[For instance, note first that the absolute error function $\left|2+\frac{2x}{q}-\left(1+\frac{1}{q}\right)^x\right|$ is an increasing function of x, the number of trials. Now, suppose that error function equals e_1, "the Error". Next, "increase a little the value of x", by say $\delta > 0$. Then we have $\left|2+\frac{2(x+\delta)}{q}-\left(1+\frac{1}{q}\right)^{x+\delta}\right| = e_2$, "the new Error". We now have two points, $(x,e_1),(x+\delta,e_2)$, where $e_1 < e_2$. Where a line through these two points intersects the x-axis is our "double false Position" (linear) approximation of the true value of x = the number of trials.]

Colin Maclaurin, 1698-1746
A Treatise of Fluxions
1742

A treatise of fluxions. Edinburgh: Printed by T. W. & T Ruddimans, 1742.

In the 18th century, Newton's method of fluxions became widely preferred among British mathematicians as an approach to the calculus, even though Newton's book was difficult. This was largely due to the appearance in 1742 of Colin Maclaurin's clear and systematic exposition, in a Euclidean spirit, of Newton's methods in his *Treatise of Fluxions*. The primary reason MacLaurin wrote his text was to answer Bishop George Berkeley's correct complaint in *The Analyst* that the new calculus had a weak foundation.

Berkeley's Analyst: http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/

748. Sir Isaac Newton's binomial theorem is of excellent uſe for extracting the roots of powers, or reducing a quantity to a ſeries of this kind; and, having made no uſe of this theorem in demonſtrating the rules in the direct method of fluxions, we may the rather give an inveſtigation of it from art. 727. Let it be required to find $\overline{1+x}^n$, where n may repreſent any integer, number or fraction, whether it be poſitive or negative. It is evident from what is ſhown in the common algebra concerning powers and their roots, that the firſt term of any power of $1+x$ is 1, and that the ſubſequent terms involve x, x^2, x^3, x^4, &c. with invariable coefficients. Suppoſe therefore $\overline{1+x}^n = 1 + Ax + Bx^2 + Cx^3 + Dx^4 +$ &c. where A, B, C, D, &c. repreſent any ſuch coefficients. By finding the fluxions (art. 727.) $n\dot{x} \times \overline{1+x}^{n-1} = A\dot{x} + 2Bx\dot{x} + 3Cx^2\dot{x} + 4Dx^3\dot{x}$ + &c. and dividing by $n\dot{x}$, we have $\overline{1+x}^{n-1} = \frac{A}{n} + \frac{2Bx}{n} + \frac{3Cx^2}{n} + \frac{4Dx^3}{n} +$ &c. And ſince this equation muſt be true, whatever the value of x may be, it follows by ſuppoſing $x = 0$, (or becauſe the firſt term of $\overline{1+x}^{n-1}$ muſt be 1) that $\frac{A}{n} = 1$. and $A = n$. By taking the fluxion of the laſt equation, $\overline{n-1} \times \overline{1+x}^{n-2} \times \dot{x} = \frac{2B\dot{x}}{n} + \frac{6Cx\dot{x}}{n} + \frac{12Dx^2\dot{x}}{n} +$ &c. and dividing by $\overline{n-1} \times \dot{x}$, we have $\overline{1+x}^{n-2} = \frac{2B}{n \times \overline{n-1}} +$

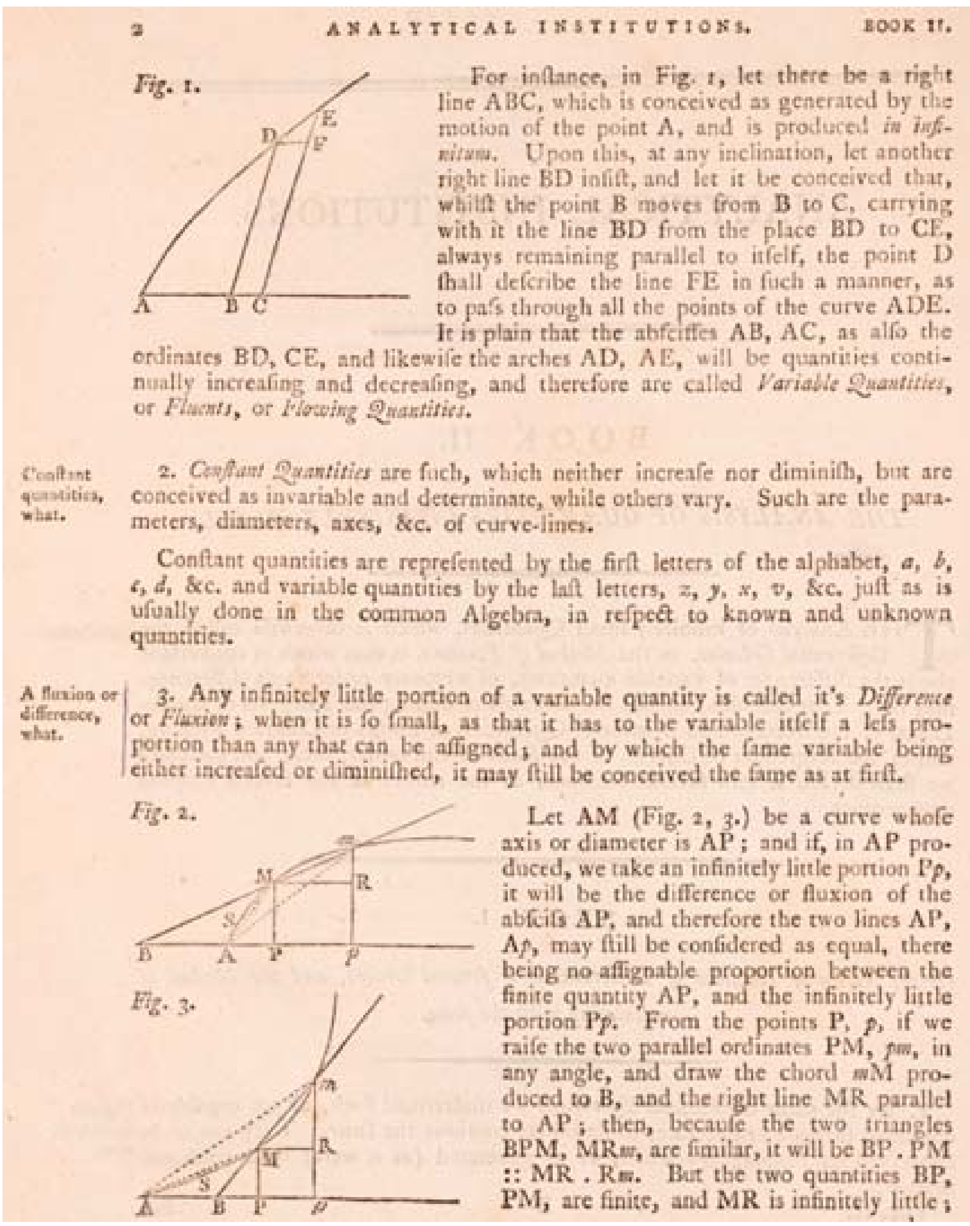
$\frac{6Cx}{n\times n-1} + \frac{12Dx^2}{n\times n-1}$ + &c. and by ſuppoſing $x = 0$, (or becauſe the firſt term of any power of $1 + x$ muſt be 1) $\frac{2B}{n\times n-1} = 1$ or $B = n\times\frac{n-1}{2}$. By taking the fluxions again we find $\overline{n-1} \times \overline{1+x}^{n-3} \times \dot{x} = \frac{6C\dot{x}}{n\times n-1} + \frac{24Dx\dot{x}}{n\times n-1}$ &c. and $\overline{1+x}^{n-3} = \frac{6C}{n\times\overline{n-1}\times\overline{n-2}} + \frac{24Dx}{n\times\overline{n-1}\times\overline{n-2}}$ + &c. ſo that $\frac{6C}{n\times\overline{n-1}\times\overline{n-2}} = 1$, or $C = n\times\frac{n-1}{2}\times\frac{n-2}{3}$; and ſo on. Therefore $\overline{1+x}^n = 1 + nx + n\times\frac{n-1}{2}\times x^2 + n\times\frac{n-1}{2}\times\frac{n-2}{3}\times x^3$ + &c.

And $\overline{a+b}^n = \frac{\overline{a+b}^n}{a^n}\times a^n = \left.\overline{1+\frac{b}{a}}\right|^n \times a^n =$ (by ſubſtituting $\frac{b}{a}$ for x) $a^n + \frac{na^nb}{a} + n\times\frac{n-1}{2}\times\frac{a^nb^2}{a^2} + n\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{a^nb^3}{a^3}$ + &c. $= a^n + na^{n-1}b + n\times\frac{n-1}{2}\times a^{n-2}b^2 + n\times\frac{n-1}{2}\times\frac{n-2}{3}\times a^{n-3}b^3$ + &c. which is the binomial theorem.

Maria Gaetana Agnesi, 1718-1799
Foundations of Analysis for the use of Italian Youth
1748

Analytical institutions. London: Printed by Taylor and Wilks, 1801.

In an age when few women participated in science and mathematics, Maria Agnesi excelled. Her greatest achievement was an exceptionally clear two volume synthesis and textbook of the calculus published in the original Italian edition in 1748. [Curiously, the influence of Newton's "dot" notation for the calculus was so dominant in England, that when John Colson translated her book into English, he changed all her Leibniz notation into Newton notation, so that only the Italian edition reflects Agnesi's true notational choices.]



2 ANALYTICAL INSTITUTIONS. BOOK II.

Fig. 1.

For instance, in Fig. 1, let there be a right line ABC, which is conceived as generated by the motion of the point A, and is produced *in infinitum*. Upon this, at any inclination, let another right line BD insist, and let it be conceived that, whilst the point B moves from B to C, carrying with it the line BD from the place BD to CE, always remaining parallel to itself, the point D shall describe the line FE in such a manner, as to pass through all the points of the curve ADE. It is plain that the abscisses AB, AC, as also the ordinates BD, CE, and likewise the arches AD, AE, will be quantities continually increasing and decreasing, and therefore are called *Variable Quantities*, or *Fluents*, or *Flowing Quantities*.

Constant quantities, what.

2. *Constant Quantities* are such, which neither increase nor diminish, but are conceived as invariable and determinate, while others vary. Such are the parameters, diameters, axes, &c. of curve-lines.

Constant quantities are represented by the first letters of the alphabet, *a*, *b*, *c*, *d*, &c. and variable quantities by the last letters, *z*, *y*, *x*, *v*, &c. just as is usually done in the common Algebra, in respect to known and unknown quantities.

A fluxion or difference, what.

3. Any infinitely little portion of a variable quantity is called it's *Difference* or *Fluxion*; when it is so small, as that it has to the variable itself a less proportion than any that can be assigned; and by which the same variable being either increased or diminished, it may still be conceived the same as at first.

Fig. 2.

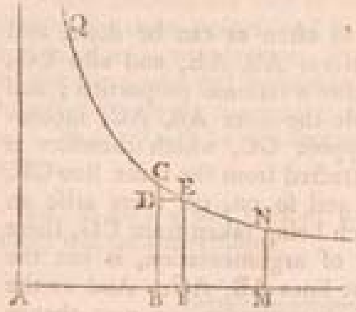
Fig. 3.

Let AM (Fig. 2, 3.) be a curve whose axis or diameter is AP; and if, in AP produced, we take an infinitely little portion P*p*, it will be the difference or fluxion of the absciss AP, and therefore the two lines AP, A*p*, may still be considered as equal, there being no assignable proportion between the finite quantity AP, and the infinitely little portion P*p*. From the points P, *p*, if we raise the two parallel ordinates PM, *pm*, in any angle, and draw the chord *m*M produced to B, and the right line MR parallel to AP; then, because the two triangles BPM, MR*m*, are similar, it will be BP . PM :: MR . R*m*. But the two quantities BP, PM, are finite, and MR is infinitely little;

then alſo Rm will be infinitely little, and is therefore the fluxion of the ordinate PM. For the ſame reaſon, the chord Mm will be infinitely little; but (as will be ſhown afterwards,) the chord Mm does not differ from it's little arch, and they may be taken indifferently for each other; therefore the arch Mm will be an infinitely little quantity, and conſequently will be the fluxion or difference of the arch of the curve AM. Hence it may be plainly ſeen, that the ſpace PMmp likewiſe, contained by the two ordinates PM, pm, by the infiniteſimal Pp, and by the infinitely little arch Mm, will be the fluxion of the area AMP, comprehended between the two co-ordinates AP, PM, and the curve AM. And drawing the two chords AM, Am, the mixtilinear triangle MAm will be the fluxion of the ſegment AMS, comprehended by the chord AM, and by the curve ASM.

4. The mark or characteriſtic by which Fluxions are uſed to be expreſſed, is by putting a point over the quantity of which it is the fluxion. Thus, if the abſciſs AP $= x$, then will it be Pp or MR $= \dot{x}$. And, in like manner, if the ordinate PM $= y$, then it will be Rm $= \dot{y}$. And making the arch of the curve ASM $= s$, the ſpace APMS $= t$, the ſegment AMS $= u$, it will be Mm $= \dot{s}$, PMmp $= \dot{t}$, AMm $= \dot{u}$. And all theſe are called *Firſt Fluxions*, or *Differences* of the *firſt Order*. And it may be obſerved, that the foregoing fluxions are written with the affirmative ſign + if their flowing quantities increaſe, and with the negative ſign − if they decreaſe. Thus, in the curve NEC, (Fig. 4.) becauſe AB $= x$, BF $= \dot{x}$, BC $= y$, it will be DC $= -\dot{y}$, the negative fluxion of y. How fluxions are repreſented, and what are their ſeveral orders.

Fig. 4.



That theſe differential quantities are real things, and not merely creatures of the imagination, (beſides what is manifeſt concerning them, from the methods of the Ancients, of polygons inſcribed and circumſcribed,) may be clearly perceived from only conſidering that the ordinate MN (Fig. 4.) moves continually approaching towards BC, and finally coincides with it. But it is plain, that, before theſe two lines coincide, they will have a diſtance between them, or a difference, which is altogether inaſſignable, that is, leſs than any given quantity whatever. In ſuch a poſition let the lines BC, FE, be ſuppoſed to be, and then BF, CD, will be quantities leſs than any that can be given, and therefore will be *inaſſignable*, or *differentials*, or *infiniteſimals*, or, finally, *fluxions*.

Thus, by the common Geometry alone, we are aſſured that not only theſe infinitely little quantities, but infinite others of inferior orders, really enter the compoſition of geometrical extenſion. If incommenſurable quantities exiſt in Geometry, which are infinites in their kind, as is well known to Geometricians

How I use these books, and their translations, for my History of Mathematics class at UMKC

If the use of the history of mathematics in the classroom is to be more than a collection of generic and occasionally entertaining stories, **as instructors we need to dig deeper**. We need to look at the actual historical written work of mathematicians and teachers of mathematics to see how they were thinking in the context of their times. Luckily we now have access to many excellent English translations of historical mathematics, and though as working teachers we don't have time to do a comprehensive archaeological excavation into them, we can dig "**test-pits**" to dip into that rich past.

The struggles of the historical mathematics research community toward understanding, when it first encountered the very same issues that our students now perennially face in learning elementary mathematics, can engage those students, if we take advantage of the universal human **pleasure in detective work**. I present my students with copies of historical arguments, problem solutions, or proofs (with some guiding notes), and often tell them little (until after the assignment) about the author, or his or her time period. So, they are faced with extracting meaning from material that is well within their grasp, but unusual in presentation. With the power of modern notation at their fingertips, along with the hundreds and sometimes thousands of years of mathematical sophistication since the material was written, they successfully learn to read with precision and to **explicate** the given arguments and proofs, and in the process build confidence in their own work. They even find this exciting, especially when I reveal who the author is.

Explicate, verb, [Definition, Oxford English Dictionary]:

1.a. To unfold, unroll; to smooth out (wrinkles);
to open out (what is wrapped up): to expand…

c. To spread out to view, display.

2. a. To disentangle, unravel.

3. To develop, bring out what is implicitly contained in (a notion, principle, proposition.)

4. To unfold in words; to give a detailed account of …

6. To make clear the meaning of (anything); to remove difficulties or obscurities from; to clear up, explain.

Historical "Proof" Explication

- Read the given argument, proof, or theorem and proof combination. I have photocopied them from an original historical document, or faithful English translation. The assignment is designed to be more-or-less self-contained.

- **Explicate this result**, that is, to write an expository version. Your version will usually therefore be longer than the original. Remember that **a "proof" is a narrative**, telling the story of (proving) why the theorem is true. Your job is to make that story transparent.

- **Stay as close as possible to the style and form of the argument**, preserving the historical flavor and ideas of the author. Do not substitute a faster, modern statement and proof.

- You will be graded on the **clarity** of your exposition.

- You will also be graded on **how critically you have read the result**, whether you found all the confusions, omitted arguments, and so on, even if you were not able to settle all of them to your satisfaction.

- Your work may require any or all of the following:

  - **Clarify** words, definitions, and statements. For instance, "line" may be used where "line segment" is meant, "equation" confused with "expression", or "equal" with "congruent" or "equivalent"; the same letters or words may be used for several different objects; out-of-date terminology and phrasing may need to be updated, or just made more precise.

  - **Is the result properly stated** as a Theorem, Proposition, Lemma, Corollary, etc.? Is the Proof so named, and clearly delineated?

  - **Add as many pictures as you like** to clarify the argument. These include "idea" pictures, as well as the usual graphs, diagrams, constructions, etc. A detailed "movie" of images is often needed.

  - **Include omitted arguments, or other details**. Some arguments may be long enough to be stated (by you) separately as a Lemma. Do so, if you like. Other arguments may be assumed common knowledge by the author, but not clear to you or your modern readers. Tell us. This is vital to good exposition.

  - **Correct any mathematical errors or omissions** you may find. For example, if a variable suddenly appears in a denominator, did the author consider the case when that variable might be zero? Are there other omissions of cases we would today include? Are there typographic errors? Are the calculations really correct? Take nothing for granted.

  - Modernize the mathematical notation if needed, but again, stay close to the history.
