

7 January 2006 – latest revision

Richard Delaware [delawarer@umkc.edu]

Some Typographical and Other Errors Noted

Elementary Real Analysis,

by Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, 2001

Note: These cover all the Chapters and sections for a first and second analysis course (Advanced Analysis I & II, Math 402 & 412) at UMKC since Winter 2003 semester.

Preface

p. xv, line 2: Delete one “the”.

Chapter 1

1.6, p.11, top, numbers 1,2,3: This is the first use of the infinity symbol and no explanation is given. [I did find this discussed on p. A-2, and in 5.1.5, p.203, Note.]

1.7, p.14, Th. 1.11, Proof, lines 4-5: You use “integer” in place of “natural number” as used in the statement of the theorem. OK, but confusing to students. This is likewise done in points 1 and 2 below.

1.8, p.16, Th. 1.13, Proof, line 4: Same as last note.

1.11, p.22, 1.11.6, last line: n should start with 0 not 1.

Chapter 2

2.2.1, p.27, line 4: The superscript for the n th term should be $n-1$, to match the formula below in line 8.

2.2.1, p.27, Iteration, line 4: This is the first introduction of the sequence notation without explanation.

2.3, p.30, Example 2.3, line 7: Poor English: “a various rank”.

2.4, p.35, Th. 2.8, statement: The second limit symbol should match the first.

2.5, p.37, Def. 2.9: You write “ $s_n > M$ ”, while in Example 2.10, last line on the page, you argue with \geq .
Confusing to students.

2.6, p.40, Example 2.13, line 3: One “+” is missing and there’s one too many ellipsis’.

2.8, p.51, Exercise 2.8.5: A period is missing before “Show”.

2.10, p.59, paragraph 4 (beginning “Choose an integer N ...”): There are 5 inequalities here. In inequalities 1, 2, and 3, on the right side, delete the parentheses around the single N . In inequalities 4 and 5, in each of the three appearances of an ellipsis, a “+” is missing.

Chapter 3

- 3.1, p.77, line 3 from the bottom: In the last limit, either replace the 1 by 2, or the superscript for 2 should be n , not $n+1$.
- 3.2, p.79, line 8 from the bottom: In the second sum, the i should be a j under the summation symbol.
- 3.2, p.81, line 6: A “+” is missing from the ellipsis.
- 3.2, p.81, Summation by Parts, line 4: A “+” is missing from the ellipsis.
- 3.2, p.83, 3.2.13: A “+” is missing from the ellipsis, and there is one too many ellipsis’.
- 3.2, p.83, 3.2.14: There is one too many ellipsis’.
- 3.3, p.85, Example 3.3, line 6: A “+” is missing from the ellipsis.
- 3.3, p.85, Example 3.3, line 9: A “+” is missing from the ellipsis. Also, the second to last $<$ should be an $=$.
- 3.3, p.85, Example 3.3, lines 10-11: First, I would replace J by K and j by k here. Otherwise you conflict with the use of J in the remainder of this example, as well as in Def. 3.2 above. Second, the expression $j > N$ needs an absolute value, becoming (with the change to k), $|k| > N$.
- 3.3.1, p. 86, Th. 3.4, Proof, line 5: After “Let” insert the phrase “ J be finite and”.
- 3.3.1, p. 87, line 13: At the end of this line insert “ $|c_N - c| < 1/N$ and”.
- 3.3.1, p.87, line 17 (last summation before the end of the proof): Delete the subscript n .
- 3.4.2, p.94, line 5 from the bottom: There should be a + and an ellipsis following $8/8^p$, and the last inequality is an $=$.
- 3.4.2, p.95, Exercise 3.4.1: The index of summation should be n .
- 3.5.3, p.101, Th. 3.14, Proof, line 5: For consistency, the first inequality $N < n$ should be \leq .
- 3.5.3, p.102, line 1: Delete the absolute value lines in the first summation.
- 3.5.3, p.102, Example 3.17, line 6: A “+” is missing from the ellipsis.
- 3.5.3, p.103, 3.5.13, line 2: The last subscripts of the a ’s and b ’s should be n not m .
- 3.6.8, p.115, line 3: The k values are finite in number and should read: $k=2,3,4,\dots,n$.
- 3.7, p.129, Example 3.47, line 4: A “+” is missing from the ellipsis.
- 3.7, p.129, line 1 from the bottom and line 3 from the bottom: A “+” is missing from each ellipsis.
- 3.7.2, p.132, line 11 from the bottom: A “+” is missing from the last ellipsis.
- 3.7.2, p.132, line 10 from the bottom: A “ $<$ ” is missing from the ellipsis.
- 3.7.2, p.132, line 9 from the bottom: In the second p term, replace the sub-subscript 1 by k .
- 3.7.3, p.134, 3.7.1: A “+” is missing from the both ellipsis’.
- 3.8.2, p.139, last line of the page: A “+” is missing from the ellipsis.
- 3.9, p.141, line 7 from the bottom: A “+” is missing from the ellipsis.
- 3.9.1, p.143, last line of the page: A “+” is missing from the ellipsis.
- 3.9.2, p.146, Example 3.59, line 2: A “+” is missing from the ellipsis.
- 3.9.2, p.147, 3.9.2 and 3.9.3: A “-” is missing from both ellipsis’.

Chapter 4

- 4.3.2, p.170, line 14: Shouldn’t this be a sequence, not a union at this time?
- 4.3.2, p.170, line 19: In part (ii), “interval” should be plural.
- 4.4, p.175, line 3: Delete “ a ”.
- 4.4, p.175, line 5: Insert “ z ” after “and” near the end of the line.
- 4.5, p.177, Example 4.20, line 10: “conclusion” needs a question mark.
- 4.5.1, p.178, Th. 4.21, Proof, paragraph 2, line 4: Delete “above”.
- 4.5.1, p.178, last line on the page: Add “for all n ”.
- 4.5.3, p.182, Proof of Cousin’s Lemma, paragraph 3, line 2: Each $\delta(z)$ should be divided by 2, since δ in the statement of the Lemma is the total length of the interval.
- 4.5.4, p.185, line 13: Insert a comma before the last N .
- 4.5.4, p.185, line 21: The subscript of V should be $i+1$. Also, before “Then”, a sentence (like that in the previous paragraph) is missing: “Finally, let U be the collection of all the intervals U_i, V_i for $i=1,2,3,\dots$ ”
- 4.5.4, p.185, line 23: Again, insert a comma before the last N .
- 4.5.4, p.186, line 3: The subscript of U should be x_i not just i .
- 4.6, p.189, paragraph 2, line 4: Insert a comma after the n th term.
- 4.7, p.190, 4.7.4, lines 3-4: There’s an unnecessary line break.

Chapter 5

- 5.1.1, p.195, Figure 5.1: The x 's should be x_0 's.
- 5.1.1, p.195, last line of the page: Insert parentheses around $10x-11$ for consistency.
- 5.2.2, p.206, Th. 5.11, Proof, line 3: Replace "E" by "the domain of f ".
- 5.2.3, p.209, Exercise 5.2.12, line 6: Place a period after the first δ .
- 5.2.6, p.216, Polynomials, line 2: A "+" is missing from the ellipsis.
- 5.3, p.222, Def. 5.26: Shouldn't the parenthetical title read (Lim Sup and Lim Inf) ?
- 5.4.2, p.228, line 9 from the bottom: Doesn't the end-of-example triangle mark belong on the next page?
- 5.4.2, p.229, line 3 before the Exercises: The first $<$ should be $=$.
- 5.4.2, p.229, last line of the page: Add a question mark.
- 5.4.4, p.232, Theorem 5.36: I think " $f^{-1}(V)$ intersect A " would be preferable to just $f^{-1}(V)$.
- 5.4.4, p.233, line 8: Delete the second (right hand) parenthesis after V .
- 5.4.4, p.235, Exercise 5.4.32: This is identical to Exercise 5.10.14 on p.252. So is it Challenging or not?
- 5.6, p.238, Th. 5.47, Proof, paragraph 3, lines 1-2: Change the last part of the sentence to read "to obtain convergent subsequences."
- 5.6, p.239, Exercise 5.6.6: This is false. A hypothesis is missing. Perhaps assume " $\text{dist}(X_i, X_j) > 0$ for all i, j ".
- 5.8, p.242, Th. 5.52, Proof, line 2 from the end of the proof: Replace the second "for" by "all".
- 5.8, p.243, 5.8.9: This problem just repeats 5.8.2 on the previous page. (This also affects the reference in 7.2.3, p.293, paragraph 2, line 4, and the answer to 5.8.9 on p.A-30.)
- 5.9.1, p.244, Jump Discontinuities, line 5: A right hand parenthesis is missing.
- 5.9.1, p.244, Essential Discontinuities, line 5: Again a right hand parenthesis is missing.
- 5.9.1, p.245, line 5: The jump should be -2 not 2 , to agree with the definition on p.244.
- 5.9.2, p.246, last word of both Def. 5.56 and 5.58: Shouldn't "on I " end both defs, as it does the others?
- 5.9.2, p.246, last line of page: The subscript should be N , not k .
- 5.9.2, p.247, Th. 5.60, last line of statement: Insert "a" after "of".
- 5.9.2, p.247, Th. 5.60, Proof, line 3: Write " $I=[a,b]$ " since " I " is not mentioned in the theorem statement.
- 5.9.3, p.250, lines 3,5,8, and 10: The four arguments for the function f should be y , not x .

Chapter 6

- 6.4.1, p.258, Example 6.7, line 8: Delete the brackets around q_1 .
- 6.4.2, p.260, Exercise 6.4.5, line 2: the subintervals (c',d') are missing their commas.
- 6.5.1, p.263, Figure 6.1: One line segment in K_2 is missing.
- 6.5.3, p.267, line 8: Delete the -1 .
- 6.7.1, p.276, line 9: Delete the subscript of 0 in the oscillation.
- 6.7.2, p.278, paragraph 3, line 4: I would add the words " f on" after the word "of".
- 6.8, p.281, Example 6.32, end of penultimate line: Delete the period; follow the comma by the word "the".

Chapter 7

- 7.2, p.286, paragraph 2, line 2: The second point is missing a right hand parenthesis.
- 7.2.3, p.293, paragraph 2, line 5: This is the first appearance of this notation for the length of an interval.
(The only previous note on length seems to be p. 279, last line.)
- 7.3.1, p.295, Th. 7.7, (i): The two x 's need the subscript 0.
- 7.3.2, p.300, equation (6): Why use the "X" multiplication symbol? You didn't use it in equation (5), p.299.
- 7.3.2, p.301, line 5: The word "than" should be "that".
- 7.3.2, p.301, 7.3.11, line 2: Delete the subscript n .
- 7.5, p.307, Th. 7.18, Proof: The definition for a local extremum first appears here.
- 7.6.1, p.310, Th. 7.19, Proof, line 6: I believe (a,b) should be $[a,b]$.
- 7.6.1, p.310, Th. 7.19, Proof, line 7: The logic dictates you should move to the end of the proof the sentence
"By Theorem 7.18, $f'(c)=0$."
- 7.7, p.316: Def. 7.22 here repeats Defs 5.55-5.58 on p.246. This note would be helpful there too.
- 7.7, p.318, 7.7.6, upper right entry of determinant: Delete the extra right hand parenthesis.
- 7.10, p.327, line 4: Add a period before "Thus".
- 7.10, p.330, 7.10.15, line 2: Insert a comma before the n th alpha term.
- 7.11.1, p.333, line 2 from the bottom of the proof: The middle term needs primes top and bottom.
- 7.11.1, p.333, line 2 from the bottom of the page: In both limits x should approach a , not 0.
- 7.11.3, p.336, line 7 from the bottom of the page: Delete the primes from top and bottom.
- 7.12, p.340, Th. 7.43, Proof, line 4: Replace $n!$ by $(n+1)!$
- 7.12, p.340, line 5 from the bottom of the page: Delete the last word of the line, "the".

Chapter 8

- 8.1, p.347, line 12: The ellipsis is missing a comma.
- 8.1, p.347, line 13: Possibly insert "circumscribed" before "rectangles". Also, the phrase "with vertices at these points" is unclear. (Height of rectangle determined by right hand endpoints?)
- 8.1, p.347, line 15: The parentheses are not needed about (k) .
- 8.2, p.350, line 17: The statement " $0 < M(\pi) - m(\pi)$ " is only true if f is nonconstant. Mention this at the start of the proof.
- 8.2.1, p.352, 8.2.7, line 2: A comma is missing before the n th interval.
- 8.2.1, p.353, 8.2.11: A "+" is missing from the ellipsis.
- 8.3, p.356, Theorem 8.8, line 1: Delete the "BST WARN" goof.
- 8.6.1, p.367, Example 8.14, end of paragraph 1: Add "and $f(0)=0$ " to complete the definition.
- 8.6.1, p.367, Example 8.14, part (i): Add the case of the k th point = 0.
- 8.6.2, p.369, line 4: Add parentheses around the (5).
- 8.6.3, p.372, line 4 from the bottom of the page: Replace $<$ by $=$.
- 8.6.4, p.373: In the statements of Corollaries 8.18, 8.19, 8.20, after the word "interval" insert " $[a,b]$ ".
- 8.7, p.375, line 7 from the bottom of the page: Delete one "on".
- 8.8, p.379, line 5 before Def. 8.29: Add parentheses around $1 - 1/X$.
- 8.9, p.381, line 9: The ellipsis is missing two $<$ symbols.

Chapter 9

- 9.2, p.387, Example 9.5, line 4: The "lim" symbol is missing before x^{n-1} .
- 9.3.1, p.395, Theorem 9.14, line 1: The subscript of the sequence should be k , not n .
- 9.5.2, p.411, part (iii), line 5: Following $L_N(x)$, replace the $+$ by $-$.
- 9.5.3, p.413, Theorem 9.33, line 5: Add something like "and all n ".

Chapter 10

- 10.3, p.433, third line from the bottom: Delete the "s" in the second appearance of "converges".
- 10.4.1, p.436, Theorem 10.11: First, delete the period at the end of expression (3). Second, the statement needs tweaking. It reads as though f is only defined on the stated open interval. So, if the interval of convergence I in fact includes either endpoint, how can f be continuous on I , since it (apparently) is not defined at the endpoints.
- 10.5.2, p.448, line 4: A - is necessary in front of M .
- 10.5.2, p.448, line 5: Add a period at the end.
- 10.6.1, p.451, Theorem 10.32: The statement is missing a reference for the use of δ .
- 10.6.1, p.451, Exercise 10.6.3, line 1: Delete the "t" at the end of "several".
- 10.8.1, p.455, line 4: You say "orthogonality" formulas here; on p.456, Exercise 10.8.3, you say "relations".
- 10.8.4, p.460, Proof, lines 7 & 8: Twice, the function is "F" not "f".

Chapter 11

- 11.2, p.464, Definition 11.3, line 4: The second x should be bold.
- 11.2, p.464, Definition 11.4, line 2: The first x and y should both be bold.
- 11.2, p.465, Figure 11.1: The first diagram should read (in bold) " $y-x$ " not " $x-y$ ".
- 11.2, p.465, Proof, 4th line from the bottom: On the right side of the inequality, delete both squares.
- 11.2, p.466, Theorem 11.6, part (i), the last 0 should be bold.
- 11.2, p.466, Proof of Theorem 11.6, line 3; In the last expression, delete both internal squares.
- 11.2, p.467, Exercise 11.2.6 (c): The first x should be bold.
- 11.2, p.467, Exercise 11.2.8 (a): The x and y should both be bold.
- 11.3, p.468, paragraph 2, line 1: The R needs a superscript of " n ".
- 11.3, p.468, paragraph 2, line 1, definition (3): delete " E " and the subset symbol, since x_0 need not be in E .
- 11.3, p.468, paragraph 2: This is the first instance of what I consider an error appearing throughout this chapter, repeatedly on nearly every page. I believe the subscript of the vectors x_0 should NOT be bold, since the subscript is not itself a vector. This was very confusing to my students, since there are times when you also use a non-bolded italic subscript k , which is correct. There are so many of these instances, that I will only indicate a few of further interest below.
- 11.4, p.472, line 8: At the end of the line k should approach infinity, not 0.
- 11.4, p.472, Theorem 11.16, line 1: Here the two sequences are correctly written, with the italic subscript.
- 11.4, p.473, Proof of Theorem 11.19, line 5: The sequence subscript is " k_1 " (the number one), not " k_j ".
- 11.4, p.473, Proof of Theorem 11.19, line 4 from the bottom: The sequence of x -values needs brackets.
- 11.4, p.474, Proof of Theorem 11.19, line 4: The sequence needs a left bracket.
- 11.5.1, p.476, Example 11.21, line 4: I believe for the closed unit ball you need either the notation not introduced until p.590, #2, or the closure notation first appearing p.469, Exercise 11.3.2 (b).
- 11.6.1, p.481, Lemma 11.30, line 3, last symbol: The y should be bold.
- 11.6.1, p.481, Proof of Lemma 11.30, line 2, last inequality: The subscript should be 0 not k .
- 11.6.1, p.481, Proof of Lemma 11.30, line 4: The first x needs a subscript k .
- 11.6.1, p.481, Proof of Lemma 11.30, line 3 from the bottom: The x needs a subscript k .
- 11.6.1, p.481, Example 11.31, lines 5 & 6: The first " $x^2 + y^2$ " should NOT have square root, while the second appearance should. (I assume the first inequality is an immediate application of the geometric mean - arithmetic mean inequality to the numbers x^2 and y^2 .)
- 11.6.1, p.482, Example 11.32, last line: The epsilon should be a delta.
- 11.6.1, p.482, Example 11.33, line 1: The second appearance of " f " should be bold.
- 11.6.2, p.483, paragraph 2, line 3: An absolute value vertical line is missing.
- 11.6.2, p.483, paragraph 2, line 6: After "while" the (x,y) should NOT be bold.
- 11.6.2, p.484, Lemma 11.35: Before "all" the word "for" is missing. Also the statement should begin by stating what f , E , and x_0 are, similar to that of Theorem 11.36 on the same page.
- 11.6.2, p.484, Theorem 11.36, line 2: The last subscript in the y -vector is " m " not " n ".
- 11.6.2, p.484, Proof of Theorem 11.36, last line: The y needs a subscript "0".
- 11.6.2, p.485, Proof of Theorem 11.36, line 2: The y needs a subscript " j " not " i ".
- 11.6.3, p.485, Proof of Theorem 11.38, line 2: The last subscript of z is in the wrong font.
- 11.7, p.488, Exercise 11.7.9, line 2: See my 11.5.1 note above about closed ball notation.

Chapter 12

- 12.2.1, p.497, line 1: The first R (Real Numbers) is in the wrong font.
- 12.2.3, p.503, Proof of Theorem 12.5, last line, Expression (11): The x should be a y.
- 12.2.3, p.504, Proof of Theorem 12.5, line 4 from the bottom, sentence 2: Both limits should approach (x_0, y_0) not $(0, 0)$.
- 12.3, p.508, line 6: This might just be a matter of taste, but if the calculations are carried out, each of the two deltas should be divided by the square root of 2.
- 12.3, p.509, line 6 from the bottom of the page: Delete the final period.
- 12.4.2, p.514, line 4: The word "as" should be "is".
- 12.4.2, p.515, Definition 12.16, part 2, line 3: Absolute values are introduced here for the h-values, but were not previously used in Definition 12.13, p.512, for the h and k. This is bothersome.
- 12.4.3, p.517, line 3: Shouldn't the epsilon function be in absolute value too?
- 12.4.4, p.518, last line and lines 6 & 7 from the bottom: Since the gradient is a vector, in these three cases there should be double bars about the gradient.
- 12.4.4, p.518, line 4 from the bottom: The zero should be bold, or, written $(0, 0)$.
- 12.4.4, p.519, Theorem 12.19, line 4, last symbol: Add double bars around the gradient.
- 12.4.6, p.523, line 6: The reference should be to expression (31) not (30).
- 12.5.5, p.535, Proof of Theorem 12.35, line 3: The "F" in the first word "Function" should not be cap.

Chapter 13

- 13.2, p.577, Example 13.5, line 3: Three commas are missing.
- 13.3.1, p.581, Example 13.9, line 2: A comma is missing in the y vector.
- 13.3.2, p.584, Exercise 13.3.3, line 2: A comma is missing in the y vector.
- 13.3.2, p.584, Exercise 13.3.3, last line: The subscript on l should be infinity.
- 13.4, p.585, lines 2 & 3: The subscript of x should be k not n in both cases.
- 13.6.1, p.600, Example 13.29, line 5: Delete the i before the n in the limit.
- 13.6.1, p.601, Example 13.31, line 2 from the bottom: Add the subscript k to the first f.
- 13.12.1, p.643, Line 2: Replace N by n in $d(x_n, x_m)$.
- 13.12.1, p.643, Line 3: Replace M by m.
- 13.12.3, p.647: All the x's should be k's.
- 13.12.3, p.647: In line 5 from the bottom of the proof replace = by the subset symbol.

Appendix A

- A.2.2, p.A-8, A.2.2, line 2: The ellipsis is missing a comma.
- A.8, p.A-16, lines 2 and 5: A "+" is missing from each ellipsis.
- A.8, p.A-17, lines 1 and 3 from the bottom of the page: A "+" is missing from each ellipsis.
- A.8, p.A-18, A.8.1: A "+" is missing from the ellipsis.

Appendix B

- p.A-22, 1.11.6, line 4: n should begin with 0 not 1.
- p.A-26, 4.5.18, line 4: After the first "a", delete the x and the element symbol.
- p.A-28, 5.2.29, line 4: Insert 0 before "for all".
- p.A-28, 5.2.44, line 6: Delete the Latex /in .
- p.A-30, 5.7.5, line 2: The second interval should begin with -N.
- p.A-33, 7.3.22, line 6: The answer should read " $x^p = e^{p \ln(x)}$ ".
- p.A-33, 7.5.6, line 2: Delete the colon. Add "x:" before f(t).
- p.A-35, 7.9.1, line 4: Delete the extra final right hand parenthesis.
- p.A-36, 7.13.7, line 9: Replace the second "p" by "q".
- p.A-38, 10.5.4, part 1: Replace the first 0 by x.